Some Notes from January 23

In class we were looking for the multiplicative inverse of a complex number $z = x + yi$. We discussed more than one method to find the inverse.

- Let $a + bi$ denote the multiplicative inverse of $x + yi$, and let us figure out an expression for a and for b. This is the one we worked out in class by solving a system of linear equations.

  We wrote out the product $(x + yi)(a + bi) = (xa - yb) + (xb + ya)i$.

  Equating real and imaginary parts gives the equations $xa - yb = 0$ and $xb + ya = 1$.

  We solved these by substitution, assuming y non-zero, and obtained $a = \frac{x}{x^2+y^2}$ and $b = \frac{-y}{x^2+y^2}$.

  (If y were zero, we would have $x + yi = x \in \mathbb{R}$, and we already know the multiplicative inverse of a non-zero real number.)

- The second method is Matt's idea, written here with the terminology and notation that we are using in class.

  We want $a + bi$ so that $(x + yi)(a + bi) = 1$

  First, choose some $(a + bi)$ for which the product will be a positive real number. The complex conjugate of $x + yi$ satisfies this, since $(x + yi)(x - yi) = x^2 + y^2$, which has imaginary part zero and is positive. Now we have a product that is a positive real number. If we scale it to length (modulus) of one, then we will have a real, positive number of modulus one. The number 1 is the only positive, real number of modulus one, and this is what we want. So, for the multiplicative inverse of $x + yi$, we have $\left(\frac{1}{x^2+y^2}\right)(x - yi)$. That is, the $a + bi$ we are looking for is $\frac{x}{x^2+y^2} - \frac{y}{x^2+y^2} i = \frac{x}{|x+yi|^2} - \frac{y}{|x+iy|^2} i$.

- In class on Thursday, we will plan to talk in more detail about the polar representation of complex numbers that was mentioned by a few students today. In particular, we will use the polar representation to try to understand what multiplication of complex numbers *looks like* in the complex plane. Using this idea, you might try to understand Matt's argument in polar coordinates.