HOMEWORK 9: DUE IN CLASS ON TUESDAY, NOVEMBER 22

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

This document has two pages.

(1) One definition for an ellipse is "the set of all points, the sum of whose distances from two fixed points is a constant." Using this definition, show that the equation defining an ellipse, centered at the origin, is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(What does "centered at the origin" mean in this context?)

The labeled diagrams from Samantha's notes should be useful.

(2) A hyperbola centered at the origin can be given by an equation of the form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Consider the graph of the function

$$y = \frac{1}{x}.$$

Write the equation of the hyperbola obtained by rotating this curve, about the origin, though an angle of $-\pi/4$.

(3) Take a look at the figure here: `https://faculty.etsu.edu/gardnerr/5025/EVESfig13.bmp`.

Carefully explain how this diagram gives a proof of the Pythagorean Theorem. Does the diagram also give a proof of the converse? Explain.

(4) (a) Write three forms for the equation of a line considered in the high school curriculum: standard form, point-slope form, and slope-intercept form.

(b) For each form, describe the information obtained when the equation is written in that form. Explain how to determine the graph of a line from each of these three forms. Use specific examples if this helps.

(5) Consider the equation of the circle given by:

$$x^2 + (y-1)^2 = 25.$$

This is a translation of a circle centered at the origin. What is the translation? Explain why the graph is translated in this direction.

(6) (a) Take a look at the mathematical tasks written to assess and develop students' ability to *see structure in expressions and equations* that are available at the Illustrative Mathematics site here: `https://www.illustrativemathematics.org/HSA`.

(b) Choose one problem that you feel does a very good job of focusing on developing student understanding of the structure of expressions and equations. Carefully explain why you think it is a good problem. Describe a context in which context might you use this problem with your own students.