

### HOMEWORK 8: DUE IN CLASS ON THURSDAY, NOVEMBER 3

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

Rational functions may be viewed as functions whose domain is a subset of  $\mathbb{R}$ . They also have an interesting algebraic structure that we discussed in class. We defined the field of rational functions in analogy to our definition of rational numbers:

$$\mathbb{Q}(x) := \left\{ \frac{f(x)}{g(x)} : f(x), g(x) \text{ are polynomials over } \mathbb{Q}, g(x) \neq 0, \text{ and } \frac{f(x)}{g(x)} = \frac{h(x)}{k(x)} \text{ if } f(x)k(x) = g(x)h(x). \right\}$$

- (1) Take a look at the rational functions given below and, *using the definition above*, group together those that are equivalent to each other. *Explain your solution.*

$$\frac{1}{x}, \frac{x+1}{x-1}, \frac{x^2-1}{x^2-2x+1}, \frac{x^2}{x+1}, \frac{x-1}{x^2-x}, \frac{x^2+x}{x^2-x}$$

- (2) For each group, determine whether or not the rational functions in each group are equal as functions, rather than just equivalent as algebraic objects. Carefully explain your response.
- (3) What can you say about the graphs of two equivalent rational functions that are not necessarily equal as functions?