MATH 512 HOMEWORK 2, SPRING 2022

DUE AT THE BEGINNING OF CLASS ON MONDAY, FEBRUARY 7

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. **Solutions with little or no justification will receive little or no credit**.

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(1) Re-read the Course Policy, which is posted to the course web page. Write your understanding of the policy on using the internet to help solve HW problems.

(2) **Read through page 40 in the course textbook**.

(3) Do exercise 2.6 from the textbook, including the proof of Them 2.7.

(4) Do exercise 2.10. That is: use induction to prove the theorem, using hints in book if needed.

   Theorem 1. *For every two positive integers a and b, there exist non-negative integers q and r, $r < a$, such that $b = aq + r$.*

   (Fix a and induct on b.)

(5) Prove that the q and r in the theorem above are unique. That is, prove the following:

   Theorem 2. *Given positive integers a and b, and suppose that q and r are nonnegative integers, $r < a$, for which $b = aq + r$. Suppose also that s and t are nonnegative integers with $t < a$ for which $b = as + t$. Then $q = s$ and $r = t$.*

   It might be helpful to consider separately the cases $r \leq t$ and $r \geq t$.
   This is exercise 2.11, and you can use the hints if they are useful.

(6) Do Exercise 3.4, parts 5 - 7.

(7) Do Exercise 3.5 parts 4 - 6.