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MATH 512 HOMEWORK 1, SPRING 2020

DUE AT THE BEGINNING OF CLASS ON MONDAY, FEBRUARY 3

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. **Solutions with little or no justification will receive little or no credit**.

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(1) Read the Course Policy, which is posted to the course web page. When is HW due? What is the policy on late HW submissions?

(2) **Read pages 1-20 in the course textbook**. Section 2.1 and 2.2 consist of a review of mathematical induction. We don't plan to formally discuss mathematical induction in class, so you should read as much of this as you need.

(3) (a) Determine the natural numbers that are $(6, 9, 20)$-accessible.

(b) If there is a largest integer N that is NOT $(6, 9, 20)$-accessible, determine that integer, and prove your result. If there is no such N, explain why.

(4) Use induction to prove the theorem:

Theorem 1. *For every two positive integers a and b, there exist non-negative integers q and r, $r < a$, such that $b = aq + r$.*

(Fix a and induct on b.)

(5) Prove that the q and r in the theorem above are unique. That is, prove the following:

Theorem 2. *Given positive integers a and b, and suppose that q and r are nonnegative integers, $r < a$, for which $b = aq + r$. Suppose also that s and t are nonnegative integers with $t < a$ for which $b = as + t$. Then $q = s$ and $r = t$.*

It might be helpful to consider separately the cases $r \leq t$ and $r \geq t$.

Date: January 28, 2020.