MAT 534: HOMEWORK 4

DUE TH, SEP 25, 2014

1. Prove the lemma formulated in class: if $K, L \subset G$ are subgroups such that K normalizes L, i.e. $kLk^{-1} = L$ for all $k \in K$, then the set $KL = \{kl, k \in K, l \in L\} \subset G$ is a subgroup isomorphic to $K \ltimes L/K \cap L$.

2. Describe all Sylow 2-subgroups and 3-subgroups of D_{12} (symmetries of a regular hexagon).

3. Prove that if $|G| = 105$, then G has a normal Sylow 5-subgroup and a normal Sylow 7-subgroup.

4. Let G be a group of order p^2q, where p, q are prime, $p < q$. Assume that p does not divide $q - 1$. Prove that then G is abelian.

5. Classify all groups of order 75.

6. Classify all groups of order 20.