MAT 534: HOMEWORK 3

DUE TH, SEPT. 18

1. (a) Prove that the group of rotations of a cube is isomorphic to S_4.[Hint: it acts on the set of diagonals...] Describe the stabilizer of a vertex; of an edge.

(b) For each pair of parallel faces of a cube consider the line passing through the centers of the faces. Using that all rotations of a cube permute these lines, construct an epimorphism $S_4 \to S_3$.

2. How many ways are there to group numbers $\{1 \dots 2n\}$ into pairs? Order of pairs and order inside each pair is not important. For example, for $n = 2$, there are three ways:

$$(12)\ (34); \qquad (13)\ (24); \qquad (14)\ (23)$$

(Hint: first show that one can define a transitive action of S_{2n} on the set of all such pairings.)

3. Let $\sigma \in S_9$ be defined by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 5 & 9 & 7 & 4 & 1 & 2 & 6 \end{pmatrix}$$

(a) Find the cycle decomposition of σ. What is the order of σ?

(b) Find the sign of σ

4. Prove that the group of even permutations A_n is generated by cycles of length 3.

5. (a) Describe all conjugacy classes in S_5. How many elements are in each conjugacy class?

(b) Describe all conjugacy classes in A_5. How many elements are in each conjugacy class?