MAT 511, Handout 1

September 22, 2016

Some common tautologies

This is not the full list, only the most common ones.

(1) Modus ponens $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
(2) $((P \Rightarrow Q) \wedge (\sim Q)) \Rightarrow \sim P$
(3) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (Q \Rightarrow R)$
(4) De Morgan's laws: $(\sim(P \vee Q)) \iff (\sim P) \wedge (\sim Q)$
(5) $(\sim(P \wedge Q)) \iff (\sim P) \vee (\sim Q)$
(6) Contrapositive: $(P \Rightarrow Q) \iff (\sim Q \Rightarrow \sim P)$
(7) $(P \vee Q) \wedge \sim Q \Rightarrow P$
(8) $\sim(P \Rightarrow Q) \iff (P \wedge \sim Q)$

Some methods of proof

Direct proof: To prove $P \Rightarrow Q$, assume P; derive Q (you are allowed to use that P is true when doing this). Then you can conclude that $P \Rightarrow Q$ is true (without any assumptions!).

Indirect proof, also known as proof by contradiction: To prove that P is true, assume $\sim P$; derive from this a contradiction (you are allowed to use that P is false when doing this). Then you can conclude that P is true (without any assumptions!).

Important special case: to prove $P \Rightarrow Q$, you can assume $\sim(P \Rightarrow Q)$ (which, by (8) above, is the same as $P \wedge \sim Q$) and get a contradiction

Proof by cases: If $P_1 \vee P_2 \vee \dots$ is true, and you have proved that $P_1 \Rightarrow Q$, $P_2 \Rightarrow Q$, …, then you can conclude that Q is true.

Proofs with quantifiers

(1) De Morgans laws: $\sim\exists x\ P(x) \iff \forall x \sim P(x)$
(2) $\sim\forall x\ P(x) \iff \exists x \sim P(x)$
(3) Given that $\forall x \in M\ P(x)$ (where $P(x)$ is some statement) and that $c \in M$, we can conclude $P(c)$.
(4) If you know that $\exists x\ P(x)$ is true, you can say "let us choose an a such that $P(a)$ is true". Warning: to denote this chosen value, you should use a variable which was not used so far in your arguments!
(5) To prove $\exists x\ P(x)$, it suffices to give one example of x for which $P(x)$ is true.
(6) To prove $\forall x\ P(x)$, you have to give a proof of $P(x)$ which would work for all possible values of x. Proving it in one (or two, or five…..) special cases is not enough. Thus, a typical proof of $\forall x \in M\ P(x)$ begins: "Let x be an arbitrary element of M..."

To **disprove** $\forall x\ P(x)$, it suffices to give one example of x for which $P(x)$ is false.