MAT 511: PRACTICE FINAL EXAM

TU, DEC 6, 2016

Your name:____

(please print)

This is a practice final exam. It is longer than the actual exam will be, but otherwise, it is similar.

Unless a problem explicitly states "no explanation required", please try to write as detailed an explanation as possible. Explanations should be such that someone who does not know how to solve this problem (but knows all previous material) can follow your arguments and understand what you are doing; if in doubt, ask. **Answers without explanations will get very little partial credit!**

You are allowed to use any results discussed in class, in the textbook, or in the HW.

Notation:

$\mathbb{Z}$ — integer numbers

$\mathbb{N}$ — natural numbers, i.e. positive integers

$\mathbb{R}$ — real numbers

	1 15 pts	2 15 pts	3 10 pts	4 15 pts	5 15 pts	6 15 pts	7 15 pts	**Total**
Grade								

1. Write the following statement

"To be a member of the local school board, you must be a resident of the town and a citizen or permanent resident of the USA"

using the following elementary statements and logical connectives:

A: you can be a member of the local school board

B: you are a resident of this town

C: you are a US citizen

D: you are a permanent resident of the USA

2. For each of the statements below, (a) write its negation and (b) determine whether the statement is true or false and give a proof. Variables x, y, z take values in the set $\mathbb{R}$ of real numbers; variables m, n take values in the set $\mathbb{Z}$ of all integers.

(a) $\exists x\ \forall y\colon xy < 1$

(b) $\forall x\ \exists! y\colon xy = 1$

(c) $\forall x\ \forall y\colon\ (x > 0 \wedge y > 0) \implies \exists n : nx > y$

3. Prove that for any $n \geq 1$, we have

$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} > \frac{13}{24}$$

4. For each of the following relations on $\mathbb{R}^2$, determine whether it is an equivalence relation. If it is, describe the equivalence class containing $(1, 1)$.

(a) $(x_1, y_1) \sim (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$

(b) $(x_1, y_1) \sim (x_2, y_2)$ iff $x_1 = x_2$ or $y_1 = y_2$

5. Without using the theorems in the book, prove that if A is some set and B_i, $i = 1, 2, \ldots$ is a sequence of sets, then we have

$$A - \bigcup_{i\in\mathbb{N}} B_i = \bigcap_{i\in\mathbb{N}} (A - B_i)$$

6. (a) Let $f\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be defined by $f(x_1, x_2) = (x_1 - x_2, 3x_2)$. Is this function injective? surjective? bijective? If bijective, what is the inverse function?

(b) Let $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be defined by $f(x_1, x_2) = (x_1 - x_2, 3x_2)$. Is this function injective? surjective? bijective? If bijective, what is the inverse function?

7. Consider the coordinate plane $\mathbb{R}^2$

(a) Let us call a triangle in $\mathbb{R}^2$ integer if coordinates of all three vertices of the triangle are integer. Prove that the set of all such triangles in $\mathbb{R}^2$ is denumerable.

(b) Prove that the set of all (not necessarily integer) triangles in $\mathbb{R}^2$ is not denumerable.