MAT 511: HOMEWORK 9

DUE TU, NOV 15

1. Prove that if f, g are functions such that composition $f \circ g$ is injective, then g is injective. Show that f might not be injective.

2. Construct bijections
(a) $\mathbb{R}_{>0} \to \mathbb{R}$
(b) $(0,1) \to \{x \in \mathbb{R} \mid x > 1\}$
(c) $(0,1) \to \mathbb{R}_{>0}$
(d) $(0,1) \to \mathbb{R}$

3. Prove that if $f\colon A \to B$ is injective, and $X_1, X_2 \subseteq A$, then $f(X_1 \cap X_2) = f(X_1) \cap f(X_2)$.

4. Prove that if A, B, C are finite sets, then

$$\begin{aligned}|A \cup B \cup C| = &|A| + |B| + |C| \\ &- |A \cap B| - |A \cap C| - |B \cap C| \\ &+ |A \cap B \cap C|\end{aligned}$$

5. Consider the sequence of integer numbers $a_1 = 1$, $a_2 = 11$, $a_3 = 111$, $a_4 = 1111$, $\ldots$. Prove that in this sequence there are two numbers whose difference is a multiple of 37.

6. Let $S \subseteq \mathbb{N}$ be a set which consists of 10 two-digit integers. Prove that then it is possible to find two different subsets $S_1, S_2 \subseteq S$ such that sum of all elements in S_1 is equal to sum of elements in S_2.

7. (Extra Bonus problem) Formulate and prove an analogue of Problem 4 for four sets.