MAT 511: HOMEWORK 7

DUE TH, OCT 27

1. Which of the following relations on the set $\mathbb{R}$ are equivalence relations? For each realtion that is an equivalence relation, describe the equivalence class $[\sqrt{2}]$.

(a) $x \sim y$ if $xy \geq 0$
(b) $x \sim y$ if $x + y = 1$
(c) $x \sim y$ if $x - y \in \mathbb{Q}$ (here $\mathbb{Q}$ is the set of rational numbers)
(d) $x \sim y$ if $x^2 = y^2$.
(e) $x \sim y$ if ($x = y$ or $xy = 1$).

2. Recall the equivalence relation on the set $\mathbb{Z}$ defined by

$$a \equiv b \mod n \qquad \text{if } n|a - b$$

(it is called "congruence modulo n").

(a) Prove that if $a \equiv b \mod n$ and $c \equiv d \mod n$, then $a + c \equiv b + d \mod n$ (in other words, congruences can be added).
(b) Prove that if $a \equiv b \mod n$ and $c \equiv d \mod n$, then $ac \equiv bd \mod n$ (in other words, congruences can be multiplied).
(c) Show that it is possible that $ab \equiv 0 \mod n$, but $a \not\equiv 0 \mod n$, $b \not\equiv 0 \mod n$.

3. Let $A = \mathbb{R} \times \mathbb{R} - \{(0, 0)\}$ be the plane with the origin removed. Consider the following relation on A:

$$(x_1, y_1) \sim (x_2, y_2) \qquad \text{if } \exists t \in \mathbb{R}_{>0} \colon x_2 = tx_1, y_2 = ty_1$$

(a) Prove that this is an equivalence relation.
(b) Describe and plot on the cooridnate plane the equivalence class $[(1, 1)]$.

4. Let $A = \mathbb{Z} \times \mathbb{Z}$. Consider the following relation on A:

$$(x_1, y_1) \sim (x_2, y_2) \qquad \text{if } x_1 - x_2 \text{ is even}, \quad y_1 - y_2 \text{ is even} .$$

(a) Prove that this is an equivalence relation.
(b) Describe the equivalence class $[(1, 1)]$.
(c) How many equivalence classes are there?