MAT 511: HOMEWORK 6

DUE TH, OCT 13

1. Use induction to prove the following formulas
(a) $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$
(b) $\sum_{i=1}^{n}(3i-2) = n(3n-1)/2$
(c) $n! > 3n$ for all $n \geq 4$
(d) $5^{2n} - 1$ is divisible by 8.

2. Let the sequence a_n be defined by $a_1 = 2$, $a_{n+1} = 3a_n - 2$. Guess the formula for a_n and use induction to prove it.

3. Read he description of the game "Towers of Hanoi" (e.g., here: https://en.wikipedia.org/wiki/Tower_of_Hanoi) and use induction to prove that this game with n disks can be solved in $2^n - 1$ moves.

4. Let F_n be the Fibonacci sequence defined in class. Prove the following:
(a) For any n, $\gcd(F_n, F_{n+1}) = 1$
(b) For any n, $F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$

5. It is known that if a (real or complex) number a is a root of polynomial $p(x)$, then $p(x)$ is divisible by $x - a$ (i.e., it can be written in the form $p(x) = (x-a)q(x)$ for some polynomial $q(x)$). Use this to prove that a polynomial of degree n can not have more than n roots. Try to be as precise as possible, avoiding words like "and so on..".