MAT 511: HOMEWORK 5

DUE TH, OCT 6

1. Let the universe be the set of all real numbers. Let $A = [3, 8)$, $B = [2, 6]$, $C = (5, \infty)$. Find

(a) $A \cap B$
(b) $A \cup B$
(c) $A \cup (B \cap C)$
(d) $A - B$
(e) A^c

2. Prove that $A \cup B = B$ iff $A \subseteq B$

3. Prove that $(A - B) \cap (A - C) = A - (B \cup C)$.

4. Give a counterexample to the following statement:
If $(A \cap B) \subseteq (C \cap B)$, then $A \subseteq C$.

5. Let the family of sets $A_n, n \in \mathbb{N}$, be defined by $A_n = \left(-n, \frac{1}{n}\right)$ (here $\mathbb{N} = \{1, 2, \dots\}$ is the set of positive integers).
Find $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$.

6. Prove that if A_i, $i \in I$ is a family of sets indexed by $i \in I$, then for any set B, we have

$$B \cap (\bigcup_{in \in I} A_i) = \bigcup_{in \in I} (B \cap A_i).$$

7. Give an example of a family of subsets $A_i \subset \mathbb{Z}$ indexed by $i \in \mathbb{N}$ such that intersection of any finite collection of them is nonempty, but intersection $\bigcap A_i$ over all i is empty.