

MAT 511: HOMEWORK 4

DUE TH, SEP 29

1. Prove or disprove the following statements. The universe is the set of all real numbers.
 - (a) $\forall x \exists y : x + y < 0$
 - (b) $\exists y \forall x : x + y < 0$
 - (c) Equation $x^2 - 4x + 1 = 0$ has a unique solution.

2. Consider the following statement:

There is a smallest positive real number.

- (a) Write this statement using only quantifiers, arithmetic operations, equalities and inequalities. Use only variables from the universe of all real numbers.
 - (b) Prove or disprove this statement.
3. Let $P(x)$, $Q(x)$ be some predicates (i.e., logical statement whose truth value depends on x). Are the following two statements equivalent? If not, does one of them imply the other?

$$\forall x [P(x) \vee Q(x)] \\ (\forall x P(x)) \vee (\forall x Q(x))$$

4. Consider the following collection of statements:

- (a) Barber John shaves all men living on Main street who do not shave themselves, and only them
- (b) Barber John is a man
- (c) Barber John lives on Main street

Show that these statements lead to a contradiction.

5. Consider the following reasoning:

- (a) No stupid man is learned
- (b) Some learned persons are pious
- (c) Therefore, some pious persons are not stupid

Write these three statements in the language of formal logic, using variables with universe “all people”, quantifiers and predicates (it is up to you to introduce notation for whatever predicates you need). Give a proof of the last statement given the first two, writing the proof in as much detail as possible.