MAT 511: HOMEWORK 3

DUE TH, SEP 22

In problems 1, 2 , you need to a)write the obvious conclusion from given statements; and b)justify the conclusion, by writing a chain of arguments which leads to it. It may help to write the given statements and conclusion by logical formulas.

1. If Jack comes home late from school, it means he either had a track meet or a theater club. After a track meet, he comes home very tired. Today he came home late but was not tired. Therefore, ...

2. (a) All babies are illogical.
(b) Nobody who can manage a crocodile is despised.
(c) Illogical persons are despised.
(Remark: you can prove it without using quantifiers, in the same way we did with "All hummingbirds are richly colored..." puzzle in class).

3. Given that

$$(\sim B) \implies M$$
$$\sim L$$
$$(\sim M) \vee L$$

prove that B is true.

4. Let m, n, k be integer numbers such that m divides n and n divides k. Prove that m divides k.
(Recall that i divides j if and only if there is an integer q such that $j = qi$.)

5. Prove that if x is a real number such that $x^3+7x+11 = 0$, then $x < 0$. You can use the usual properties of real numbers, such as associative, commutative and distributive laws, basic properties of inequalities (if $a < b$, and $c > 0$, then $ac < bc$, etc).

6. Consider the following arguments:

(a) No homework is fun.
Some reading is homework.
Therefore, some reading is not fun.

(b) All informative things are useful.
Some websites are not useful.
Some websites are not informative.

For each of them, (1) write it in symbolic form, using quantifiers. Write the notation you are using (e.g., $F(x)$ for "x is fun", $H(x)$ for "x is homework", etc) and (2) prove it, using the methods of proof discussed in class.

FYI: these are examples of two of Aristotle's syllogisms, namely *Ferio* and *Baroco*. For more information, google Aristotle and syllogism.

7. A subset $A \subset \mathbb{R}$ is called bounded if there exists a real number M such that $|x| \leq M$ for all $x \in A$.

(a) Rewrite this definition using only quantifiers, logic connectives, arithmetic operations, and inequalities.

(b) Write a definition of unbounded set without using the word "not" (or symbol $\nexists$).

(c) Show that the set of negative real numbers is unbounded.