

MAT 511: HOMEWORK 11
DUE TU, DEC 6

In problems in this assignment, you can write the answers as expressions containing ${}_N C_k$, $k!$ and powers. You are not required to compute these expressions.

1. For each of the following sets, determine whether it is finite, denumerable, or neither.
 - (a) Set \mathbb{Q}_+ of positive rational numbers
 - (b) $\mathbb{Q} \times \mathbb{Q}$
 - (c) Set of all polynomials of degree $\leq N$ with rational coefficients.
 - (d) Set of all polynomials with rational coefficients.
 - (e) Set of all algebraic numbers. (A real number x is called algebraic if it is a root of a polynomial with integer coefficients.)
2. Is it true that if A, B are denumerable sets, then $A - B$ is denumerable?
3. Show that the set $\mathbb{R} - \mathbb{Q}$ of all irrational numbers is not denumerable. (Hint: use proof by contradiction.)
4. Prove that the following sets are not denumerable:
 - (a) Set of all infinite sequences of zeros and ones.
 - (b) Set of all functions $\mathbb{N} \rightarrow \mathbb{N}$
 - (c) The unit circle $S = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$