MAT 511: HOMEWORK 10
DUE TU, NOV 22

In problems in this assignment, you can write the answers as expressions containing $_NC_k$, $k!$ and powers. You are not required to compute these expressions.

1. A standard deck of cards contains 52 cards: 13 cards in each of 4 suits. A *hand* is a collection of 5 cards (order doesn't matter). Determine
   (a) Number of possible hands in which 4 cards are of one suit and the fifth card is of different suit.
   (b) Number of possible hands in which exactly one card is a heart
   (c) Number of possible hands in which 4 cards have the same rank
   (d) Number of possible hands in which exactly 3 cards have the same rank and two other cards have different ranks
   (e) Number of possible hands where the five cards are in consecutive rank. [There are different rules about where the aces should be placed in the list of ranks. For this problems, use the convention that the ranks are ordered as follows: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, K, Q, A.]

2. How many ways are there to put 15 books on 3 bookshelves? [Order on each bookshelf matters; each bookshelf can contain anywhere from 0 to 15 books.]

3. Prove that the number of ways to select an even number of objects out of 2017 is equal to the number of ways to select an odd number of objects, by constructing a bijection between these sets.

4. Prove the following identity (assuming $0 < k \leq n$):
$$k \cdot {}_nC_k = n \cdot {}_{n-1}C_{k-1}$$
in two ways:
   (a) By induction in n.
   (b) Combinatorially, by showing that the left-hand side and right-hand side represent two ways of counting elements in the same set.