MAT 319: HOMEWORK 9

DUE WED, NOV 22

1. Let f, g be such that $\lim_{x\to c} f(x) = L$, and $\lim_{x\to c} g(x) = \infty$. Show that if $L > 0$, then $\lim_{x\to c} f(x)g(x) = \infty$. Give an example showing that if $L = 0$, then it is not necessarily true that $\lim_{x\to c} f(x)g(x) = \infty$

2. Let f, g be such that $\lim_{x\to c} f(x) = L$, and $\lim_{x\to c} g(x) = \infty$. Show that then $\lim_{x\to c} f(x) + g(x) = \infty$.

3. (a) Let $f(x)$ be a continuous function such that $f(c) > 0$. Prove that then there is a neighborhood $V_\delta(c)$ such that $f(x) > 0$ for all $x \in V_\delta(c)$.

(b) Let $f(x)$, $g(x)$ be continuous functions such that at some point c, $f(c) > g(c)$. Prove that then there exists a neighborhood $V_\delta(c)$ such that $f(x) > g(x)$ for all $x \in V_\delta(c)$.

4. Let $[x]$ = largest integer $\leq x$. Find all points where the function $f(x) = [x]$ is not continuous.

5. Let $f(x)$ be a continuous function. Prove that $|f(x)|$ is a continuous function.

6. For each of the following functions, determine all points where they are not continuous:

$$\text{(a) } \frac{1}{\sin x} \qquad \text{(b) } \frac{x^2 - 3x + 2}{x^2 - 1} \qquad \text{(c) } f(x) = \begin{cases} 2x+1, & x > 0 \\ x, & x \leq 0 \end{cases}$$

7. Let $f(x)$ be a continuous function on interval $[a, b]$ such that $f(x) > 0$ for all $x \in [a, b]$. Show that then $\inf\{f(x) \mid x \in [a, b]\} > 0$. Is the same statement true if we replace closed interval $[a, b]$ by an open interval (a, b)?