MAT 319: HOMEWORK 8

DUE FRIDAY, NOV 10

1. Compute each of the following limits; if a limit doesn't exist, prove it.

(a) $\lim\limits_{x\to 3} \dfrac{x^2-4x+3}{x-3}$ (b) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$ (c) $\lim\limits_{x\to 0} \cos(1/x)$

(d) $\lim\limits_{x\to 0} x\cos(1/x)$ (e) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1+2x}}{x}$ (f) $\lim\limits_{x\to 3} \dfrac{x^2+4x+3}{x-3}$

(g) $\lim\limits_{x\to 3+} \dfrac{x^2+4x+3}{x-3}$ (h) $\lim\limits_{x\to \infty} \dfrac{x^2+4x+3}{x-3}$ (i) $\lim\limits_{x\to \infty} \dfrac{x^2+4x+3}{x^2-3}$

(j) $\lim\limits_{x\to \infty} \dfrac{e^x+1}{e^x-1}$

2. Let $f\colon A \to \mathbb{R}$ be a function and $c \in \mathbb{R}$ a cluster point of A such that the following property holds: for any neighborhood $V_\delta(c)$, there are points $x_1, x_2 \in V_\delta(c)\cap A$, $x_1, x_2 \neq c$ such that $|f(x_1) - f(x_2)| \geq 1$. Prove that then $\lim_{x\to c} f(x)$ does not exist.

3. Let f, g be such that $\lim_{x\to c} f(x) = L$, and $\lim_{x\to c} g(x) = \infty$. Show that if $L > 0$, then $\lim_{x\to c} f(x)g(x) = \infty$. Give an example showing that if $L = 0$, then it is not necessarily true that $\lim_{x\to c} f(x)g(x) = \infty$

4. Use sequential criterion to prove that if $\lim_{x\to\infty} f(x) = \infty$, $\lim_{y\to\infty} g(y) = \infty$, then $\lim_{x\to\infty} g(f(x)) = \infty$.