MAT 319/320: HOMEWORK 5

DUE FRIDAY, OCT 20

1. For each of the following sequences, find out whether it is convergent. If convergent, find the limit.

(a) $a_n = (-1)^n + \frac{1}{n}$.
(b) $b_n = \frac{\sin n}{n}$
(c) $c_n = \frac{4n^2+2n-1}{2n^2-3}$
(d) $d_n = \frac{2^n}{n!}$ (recall that $n! = 1 \cdot 2 \ldots (n-1) \cdot n$.)
(e) $e_n = \left(1 + \frac{2}{n}\right)^n$.

2. Let the sequence a_n be defined by $a_1 = 1$, $a_{n+1} = \sqrt{1 + a_n}$.

(a) Show that for any n we have $a_n < a_{n+1} < \Phi$, where Φ is the root of the equation $x = \sqrt{1+x}$.
(b) Show that a_n converges and find the limit.

3. Let the sequence x_n be defined by $x_1 = 1$, $x_{n+1} = \sin(x_n)$. Show that the sequence is convergent and find the limit.

4. Let $I_n = [a_n, b_n]$ be a sequence of nested intervals such that $\lim(b_n - a_n) = 0$. Show that then $\lim a_n = \lim b_n$, and if we denote this common limit by a, then $\bigcap_{n=1}^{\infty} I_n = \{a\}$.

5. Let a_n be an unbounded sequence.

(a) Show that there exists a subsequence a_{n_k} such that $\lim(1/a_{n_k}) = 0$.
(b) Is it true that for any subsequence a_{n_k} we have $\lim(1/a_{n_k}) = 0$?