

MAT 319/320: PRACTICE PROBLEMS FOR CHAPTER 2

Friday Oct. 6 will be the first midterm. Because of this, no homework is assigned for this week; instead, here are some practice problems for the material of Chapter 2. Try doing them and comparing your solutions to the posted ones. **These problems are optional - no need to hand them in.**

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$.
Is it true that $\inf f(x) > 0$? Give a proof or a counterexample.
2. Prove that for any $x > 0$, there exist a rational number r such that $x/2 < r < x$.
(Use the Archimedean property.)
3. Let $I_n = (0, 1/n]$ and $J_n = [1 - 1/n, 1]$ for $n \in \mathbb{N}$. Show that
 - a) $\bigcap_{n=1}^{\infty} I_n = \emptyset$;
 - b) $\bigcap_{n=1}^{\infty} J_n = \{1\}$.
4. Does the nested intervals property hold for \mathbb{Q} ? Justify your answer.
5. Find the first four digits in the binary representation of $1/7$.