

MAT 319/320: HOMEWORK 3
DUE FRIDAY, SEPT. 29

1. Prove Corollary 2.5.5.
2. Find all $x \in \mathbb{R}$ satisfying inequality $|x - 1| > |x + 1|$.
3. (a) Is there a set $X \subset \mathbb{R}$ with the following three properties:
 - (i) X has no upper bound,
 - (ii) $\sup X$ exists, but does not belong to the set X ,
 - (iii) $\inf X$ does not exist?(b) Is there a set $Y \subset \mathbb{R}$ with the following three properties:
 - (i) $\sup Y$ exists and belongs to Y ,
 - (ii) $\inf Y$ exists, but does not belong to Y ,
 - (iii) 5 is a lower bound for Y ?(c) Is there a set $Z \subset \mathbb{R}$ with the following four properties:
 - (i) 10 is an upper bound for Z ,
 - (ii) -10 is a lower bound for Z ,
 - (iii) -3 is an upper bound for Z ,
 - (iv) $\inf Z = -5$?(d) Is there a set $W \subset \mathbb{R}$ with the following three properties:
 - (i) W has no lower bound,
 - (ii) -1 is an upper bound for W ,
 - (iii) $\sup W$ does not exist?(Explain your answers.)
4. Let $S = \{\frac{1}{n^2} \mid n \in \mathbb{N}\}$. Find $\inf S$, $\sup S$ (if they exist).
5. Let X be a non-empty set. Let functions f and g be defined on X and have bounded ranges in \mathbb{R} . Show that
$$\sup\{f(x) + g(x) \mid x \in X\} \leq \sup\{f(x) \mid x \in X\} + \sup\{g(x) \mid x \in X\}.$$
Give an example of two functions for which $\sup\{f(x) + g(x) \mid x \in X\}$ is not equal to $\sup\{f(x) \mid x \in X\} + \sup\{g(x) \mid x \in X\}$ (for some set X).
What would be the corresponding inequality for infimums?
6. Let $S \subset \mathbb{R}$ be non-empty.
 - (a) Show that if $u = \sup S$, then for every $n \in \mathbb{N}$ the number $u - 1/n$ is not an upper bound of S , but the number $u + 1/n$ is an upper bound of S .
 - (b) Suppose that a number $v \in \mathbb{R}$ has the properties: (i) for every $n \in \mathbb{N}$ the number $v - 1/n$ is not an upper bound of S , and (ii) for every $n \in \mathbb{N}$ the number $v + 1/n$ is an upper bound of S . Prove that $v = \sup S$.