MAT 319: HOMEWORK 10

DUE MON, DEC 11

1. Use Intermediate Value Theorem to show that $p(x) = x^4 + 7x^3 - 9$ has at least 2 real roots. Between which integers do they lie?

2. Show that the equation $x = \cos x$ has a solution on $[0, \pi]$.

3. Let $f(x)$ be a continuous function on $\mathbb{R}$ such that $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$. Show that then $f(x)$ is bounded on $\mathbb{R}$.

4. Determine where each of the functions is differentiable.

(a) $|x| + |x + 1|$

(b) $|\sin x|$

(c) $\frac{1}{1-2^x}$

5. Show that the function $f(x) = x^2 \cos(1/x)$ is differentiable everywhere, including $x = 0$. Show that the derivative is not continuous at $x = 0$.

6. Compute the derivatives of the following functions

(a) $(x + 1)^{10}$ (b) $\sqrt{1 - x^2}$

(c) $\dfrac{1}{1 + x^2}$ (d) $\sin(1 + 2x)$

7. Given that $h(x) = x^3 + 2x + 1$ has an inverse function $h^{-1}(x)$ defined everywhere on $\mathbb{R}$, compute $(h^{-1})'(y)$ for $y = h(0)$ and for $y = h(2)$.

8. Use Mean Value Theorem to show that $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.