MAT 319: REVIEW SHEET FOR MIDTERM 2

The midterm will cover all material studied since last midterm — that is, Chapters 3 and 4 of the textbook. There will be no questions which target the material of the first two chapters — but you might still need them: for example, in order to prove results about the limit of monotone sequence, you need to uderstand what the least upper bound is.

As before, I expect both computing skills and ability to state important results and construct simle proofs. In particular, you must be able to state

- Definition of limit of a sequence, sum of an infinite series, and limit of a function.
- Statement of the theorem about convergence of monotone bounded sequence, Bolzano—Weierstrass theorem, and Cauchy criterion
- Statement of limit theorems for sequences and functions and various convergence tests for sequences (comparison, squeeze theorem, ratio test).
- Definition of cluster point for sets and sequences

There will be five short questions on the exam, each worth 10 points. **The best way to prepare for the exam is to go over all the homeworks**. On the next page, there are some additional practice problems.

Here are some sample problems for the exam. Some of the questions are too long for the actual exam, and would be shortened. There are more than 5 questions here, to give you more practice.

1. (a) State the convergence theorem for monotone sequences.
(b) Prove that if a sequence a_n is monotone increasing, and has a convergent subsequence a_{n_i}, then a_n is convergent.

2. Let a sequence a_n be defined by $a_1 = 3$, $a_{n+1} = 1 + \sqrt{a_n - 1}$. Prove that $a_n \geq 2$ for all n, and that a_n is convergent. Find the limit.

3. Are the following sequences convergent, properly divergent, neither? If they are convergent, what are their limits? Prove your claims.

(a) $x_n = \dfrac{(-1)^n n}{n+3}$ (b) $x_n = \dfrac{(-1)^n n}{n^2+3}$ (c) $x_n = \dfrac{n^2}{n+3}$

4. Is the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{2n+\sqrt{n}}$$

convergent? If it is convergent, can you find the sum?

5. Prove that if a function $f\colon \mathbb{R} \to \mathbb{R}$ has a limit as $x \to 1$, then there exists a neighborhood $V_\delta(1)$ such that $f(x)$ is bounded in this neighborhood. Is it true that $f(x)$ is bounded in any neighborhood of 1?