Midterm 1

MAT 312 - Lecture 1

Feb 27, 2014

Name: (please print)	**ID #**:

No notes or books. Calculators are allowed.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit. If the problem ask you to solve some equation, you must find all solutions; if there are no solutions, you must explain why not.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need.

Each problem is worth 10 pts.

	1	2	3	4	5	**Total**
Grade						

1. The sequence F_n of Fibonacci numbers is defined by the conditions $F_1 = 1$, $F_2 = 1$, $F_{n+1} = F_n + F_{n-1}$ for all $n > 1$. The first several Fibonacci numbers are given here:

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21\ldots$$

Prove that for any $n \geq 1$, we have

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

2. For each of the following equations, you need to find at least one solution (in integer numbers) if it exists. If not, you must explain why it doesn't exist.

(a) $234x + 498y = 18$

(b) $234x + 498y = 15$

(c) Compute least common multiple of 234, 498. [You can leave the answer as a product — no need to do the multiplication.]

3. Find all solutions of the following congruence:

$$22x \equiv 18 \mod 28$$

4. Jack got a bag of coins. After counting them, he found that if you try to divide the coins into 5 equal piles, there will be two coins left; if you try to divide them into 4 piles, one coin will be left. However, the coins can be divided into three equal piles.

(a) Let n be the number of coins Jack had. Write the given information about n as a system of congruences.

(b) What is the smallest possible number of coin Jack could have?

5. (a) Find $\phi(18)$, where ϕ is Euler's phi function.
(b) Compute $5^{2012} \mod 18$.