MAT 310 FALL 2014 REVIEW FOR MIDTERM 1

GENERAL

The exam will be in class on Th, Oct 2. It will consist of 5 problems. It will be closed book: no books or notes allowed. Calculators are allowed (but are useless). Of course, no cell phones and no laptops, tablets, or other electronic devices can be used.

The exam will cover material up to and including Section 2.6. For your convenience, a listing of topics covered and skills expected of you is given below.

MATERIAL COVERED

 $\S1.1 - 1.3$. Definition of a vector space. Basic properties such as uniqueness of zero. Examples: F^n , matrices, polynomials. Definition of a subspace. Notion of a sum of two subspaces; in the textbook, it is in exercises (p. 22).

§1.4 –1.5. Linear combinations. Subspace spanned by a given set S. Definition of linear dependence/independence. Checking whether a given set of vectors in F^n is linear independent by solving a system of linear equations. Choosing in a generating set S a linear independent subset which generates the same subspace as S.

 $\S1.6$ Definition of basis. Theorem: if V has basis of n elements, then any linearly independent subset of V contains at most n elements. Definition of dimension and independence of dimension on the choice of basis. Writing a given vector as linear combination of basis elements. Dimensions of usual vector spaces such as space of matrices. Dimension of a subspace. Subsection on Lagrange Interpolation formula in 1.6 and all of 1.7 are not included.

§2.1 Definition of linear transformation. Theorem: linear transformations form a vector space. Examples: rotations in \mathbb{R}^2 ; reflections and projections. Null space and range; nullity and rank. Relation between dim V, nullity(T) and rank(T).

§2.2-2.3. Matrix of a linear transformation in given bases of V, W. Identification $\mathcal{L}(V, W) \simeq M_{m \times n}(F)$ (depends on the choice of basis!). Composition of linear transformation and matrix multiplication. Section on application in 2.3 is not included.

§2.4. Notion of inverse linear transformation. Theorem: inverse of a linear transformation is linear. Definition of isomorphism. Theorem: two finite-dimensional vector spaces are isomorphic if and only if they have the same dimension.

§2.5 Change of coordinate matrix. Similar matrices.

§2.6 Dual vector space. Dual bases. Transpose linear transformation.