Midterm 1 Makeup

MAT 310

Oct 14, 2014

Name: (please print)	**ID #**:
Your recitation: Mon Tu (please circle)	

No books or notes; calculators are allowed. Please show your work and **provide full solutions**, with your reasoning, not just answers. Solutions must be readable; the grader should be able to follow your arguments. Cross out anything the grader should ignore: everything not crossed out will be considered to be part of your solution.

	1	2	3	**Total**
Grade				

(1) Consider the following three vectors in $\mathbb{R}^3$

$$\begin{aligned} v_1 =& (2, 3, 1) \\ v_2 =& (0, -1, 3) \\ v_3 =& (3, 3, 6) \end{aligned}$$

(a) Prove that these three vectors are linearly dependent
(b) Choose a maximal linearly independent subset of $\{v_1, v_2, v_3\}$.
(c) Extend the subset you chose in part (b) to a basis in $\mathbb{R}^3$

(2) Let P_2 be the space of polynomials with real coefficients of degree at most 2. Let $D\colon P_2 \to P_2$ be given by

$$D(p) = p' - p$$

(a) Write the matrix of R in the basis $\beta = \{x^2, x, 1\}$ of P_2.
(b) Write the matrix of R in the basis $\beta' = \{x^2, x+1, x-1\}$

(3) Let V be a vector space and let $A, B\colon V \to V$ be linear transformations such that A is invertible and $B^2 = AB$.

(a) Is it necessarily true that $A = B$? If yes, prove; if not, give a counterexample.

(b) Prove that $N(B) \cap R(B) = \{0\}$

(4) Let $T\colon V \to W$ be a linear transformation, and let $L \subset V$ be a subspace.

(a) Prove that $T(L) = \{T(v) \mid v \in L\}$ is a subspace in W, of dimension $d = \dim L - \dim(L \cap N(T))$.

(b) If $\dim V = 7$, $\dim R(T) = 4$, and $\dim L = 5$, what is the maximal possible $\dim T(L)$? What is the minimum possible dimension of $T(L)$?