MAT 310 FALL 2014
REVIEW FOR FINAL

General

The exam will be in class on Mon, Dec 15, 5:30–8pm. It will consist of 7-8 problems. It will be open book: you are allowed to use books but no notes or other materials. Calculators are allowed (but are useless). Of course, no cell phones and no laptops, tablets, or other electronic devices can be used.

Material covered

§1.1 – 1.3. Definition of a vector space. Basic properties such as uniqueness of zero. Examples: F^n, matrices, polynomials. Definition of a subspace. Notion of a sum of two subspaces; in the textbook, it is in exercises (p. 22).

§1.4 –1.5. Linear combinations. Subspace spanned by a given set S. Definition of linear dependence/independence. Checking whether a given set of vectors in F^n is linear independent by solving a system of linear equations. Choosing in a generating set S a linear independent subset which generates the same subspace as S.

§1.6 Definition of basis. Theorem: if V has basis of n elements, then any linearly independent subset of V contains at most n elements. Definition of dimension and independence of dimension on the choice of basis. Writing a given vector as linear combination of basis elements. Dimensions of usual vector spaces such as space of matrices. Dimension of a subspace. **Subsection on Lagrange Interpolation formula in 1.6 and all of 1.7 are not included.**

§2.1 Definition of linear transformation. Theorem: linear transformations form a vector space. Examples: rotations in $\mathbb{R}^2$; reflections and projections. Null space and range; nullity and rank. Relation between $\dim V$, $nullity(T)$ and $rank(T)$.

§2.2-2.3. Matrix of a linear transformation in given bases of V, W. Identification $\mathcal{L}(V, W) \simeq M_{m\times n}(F)$ (depends on the choice of basis!). Composition of linear transformation and matrix multiplication. **Section on application in 2.3 is not included**.

§2.4. Notion of inverse linear transformation. Theorem: inverse of a linear transformation is linear. Definition of isomorphism. Theorem: two finite-dimensional vector spaces are isomorphic if and only if they have the same dimension.

§2.5 Change of coordinate matrix. Similar matrices.

§2.6 Dual vector space. Dual bases. Transpose linear transformation.

§3.1, 3.2. Elementary row operations. Theorem: elementary row operations are equivalent to multiplication on the left by an elementary matrix. Row echelon form and reduced row echelon form. Rank of a matrix. Theorem: rank=max. number of linearly independent rows=max. number of linearly independent columns. Inverse of a matrix; a square $n \times n$ matrix is invertible iff it has rank n. Computing inverse by elementary row operations.

§3.3-3.4. Systems of linear equations; dimension and basis of solution space of homogeneous system. Reaction between dimension of solution space and the rank of the matrix. Testing consistency of nonhomogeneous system of linear equations. Finding maximal linear independent set of columns of a matrix.

Note: application to economics discussed in Section 3.3 (starting on page 176) is not included.

§4.1–4.3, 4.5 Determinant. (Note that the way it was present in lectures is different from the one in the textbook.) Determinant of $2x2$ matrix as oriented area of a parallelogram. Characterization of the determinant by 3 properties (linear in rows, alternating (skew-symmetric), $\det(I) = 1$). Behavior of determinant under elementary row operations. Explicit formula for the determinant as sum over all permutations.

Further properties of determinant: $\det(AB) = \det(A)\det(B)$. Theorem: A is invertible iff $\det A \neq 0$. $\det(A) = \det A^t$. Cofactor expansion. Determinant of an upper-triangular and block upper-triangular matrix (Exercise 21 in Section 4.3).

Note: Cramer rule will not be included in the final exam.

§5.1 Definition of characteristic polynomial; independence of choice of basis. Eigenvalue and eigenvector.

§5.2 Eigenspace. Relation between multiplicity of an eigenvalue and dimension of the eigenspace. Theorem: eigenvectors with different eigenvalues are linearly independent. Diagonalization. Theorem: if all multiplicities of eigenvalues are 1, then the matrix is diagonalizable. Application to computation of A^N, for large N.

§5.4 Cayley-Hamilton theorem.

§7.1 – 7.2 Generalized eigenspace. Theorem: $V = K_{\lambda_1} \oplus K_{\lambda_2} \oplus \dots$. Definition of Jordan canonical form. Statement of the theorem about existence and uniqueness of Jordan canonical form of a linear transformation (when the characteristic polynomial splits) — no proof. Examples.