Practice Final
MAT 310
Dec 2014

Name: **ID #**:
(please print)
Your recitation: Mon Tu (please circle)

Textbook is allowed; no notes or other materials. Calculators are allowed. Please show your work and **provide full solutions**, with your reasoning, not just answers. Solutions must be readable; the grader should be able to follow your arguments. Cross out anything the grader should ignore: everything not crossed out will be considered to be part of your solution.

	1	2	3	4	5	6	7	**Total**
Grade								

This practice final is expected to be harder than the real exam.

(1) Consider $S = \{[2,-3,4,-1]^t, [-6,9,-12,3]^t, [3,1,-2,2]^t, [2,8,-12,3]^t, [7,6,-10,4]^t\}$

(a) Is S linearly independent? If not, find a maximal linearly independent subset.

(b) Does S span $\mathbb{R}^4$? If not, express $span(S)$ in terms of a minimal spanning set.

(c) Construct a basis for $span(S)$. What is $\dim(span(S))$?

(d) Construct a basis for $\mathbb{R}^4$ that contains the maximal linearly independent subset found in part (a).

(2) Let A be a 5×5 matrix of rank 3.

(a) Consider the function $f_A \colon M_{5\times 5} \to M_{5\times 5}$ given by $f_A(X) = AX$. (Here $M_{5\times 5}$ is the space of 5×5 matrices.) Prove that f_A is a linear transformation. What is the nullity and rank of f_A?

(b) Prove that the set of 5×5 matrices X such that $AX = 0$ is a vector space. Find its dimension.

[Hint: a 5×5 matrix consists of 5 columns, each being a vector in $\mathbb{F}^5$.]

(3) Let A be an $n \times n$ square matrix with integer entries and such that $\det A \neq 0$.

(a) Prove that then A^{-1} has rational entries.

(b) Give an example when A^{-1} is not an integer matrix .

(c) Prove that if $\det(A) = 1$, then A^{-1} is an integer matrix.

(4) Let

$$A = \begin{bmatrix} 4 & 3 & 1 & 2 \\ 1 & 9 & 0 & 2 \\ 8 & 3 & 2 & -2 \\ 4 & 3 & 1 & 1 \end{bmatrix}$$

(a) Calculate the determinant of A using any method you know.
(b) What is the determinant of $-2A$? Why?

(5) (a) Find eigenvalues of the linear transformation $A\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by the matrix

$$\begin{bmatrix} -1 & 3 & -3 \\ 0 & 5 & -3 \\ 0 & 3 & -1 \end{bmatrix}$$

(b) Find a basis for each eigenspace of A.

(c) Is A diagonalizable?

You need to justify all of your answers.

(6) Let $T\colon V \to V$ be a diagonalizable linear transformation with characteristic polynomial $(t-1)^2(t+0.5)(t-0.1)^2$.

(a) Prove that linear transformation $R = (T + 2I)$ is invertible.

(b) Show that the limit $S = \lim_{n\to\infty} T^n$ exists.

(c) Prove that $S = \lim T^n$ is a projection operator: $S^2 = S$. Can you describe $R(S)$ (for example, if $\{v_1, \ldots, v_5\}$ is a basis of V consisting of eigenvectors of T, then can you choose some subset of $\{v_1, \ldots, v_n\}$ which would be a basis of $R(S)$?

(7) Let V be the space of polynomials of degree at most 3, and let $D\colon V \to V$ be given by

$$D(p) = p' + 2p''$$

(a) Show that D is nilpotent. What is the smallest k such that $D^k = 0$?
(b) What is the characteristic polynomial of D?
(c) What is the Jordan canonical form of D? [You do not need to find a basis, just describe what will be the blocks in the Jordan canonical form.]