

MAT 200: PRACTICE FINAL EXAM
THURSDAY, MAY 5, 2016

Your name: _____
(please print)

This is a practice final exam.

It is an open book exam: you can use your notes, homeworks and handout, and our textbook, but no other books.

Notation:

\mathbb{Z} — integer numbers

\mathbb{Z}_+ — positive integers

\mathbb{R} — real numbers

1. Let $U \subset \mathbb{R}$. A point $x \in U$ is called an *interior* point of U if there is a $\delta > 0$ such that for every $z \in \mathbb{R}$, if $|x - z| < \delta$, then $z \in U$.
 - (a) Prove that point $x = 0.5$ is an interior point of interval $(0, 1)$.
 - (b) Prove that every point $x \in (0, 1)$ is an interior point of this interval.
 - (c) Without using any negatives except \notin , write a definition of what it means for a point $x \in U$ not to be an interior point of U . You may write this symbolically or in words, as you prefer, but write it carefully and correctly.

2. Let a sequence a_n be defined by the formulas $a_1 = 1$, and

$$a_{n+1} = \frac{1}{2} \left(a_n + \frac{3}{a_n} \right)$$

Prove that then for all $n \geq 1$ we have $1 \leq a_n \leq 3$.

3. Consider the sequence of integer numbers 1, 11, 111, 1111, Prove that this sequence will contain two numbers whose difference is a multiple of 2017.

4. Let \mathbb{R}_+ denote the set of positive real numbers. Let $f: \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}$ be given by $f(x, y) = x/y$.
- (a) Is f an injective function? Prove your answer.
 - (b) Is f a surjective function? Again, prove your answer.
 - (c) Is f a bijection? Prove your answer.

5. Consider the relation on \mathbb{R}^2 defined by $(x_1, y_1) \sim (x_2, y_2)$ if there exists a positive real number t such that $x_2 = tx_1$, $y_2 = y_1/t$. Is it an equivalence relation? If it is, what is the equivalence class of $(1, 1)$?

6. Find the last digit of 3^{2017} .