MAT 200: HOMEWORK 11

DUE WED, MAY 4, 2016

Notation $\mathbb{Z}_n$ used in the book is the same thing as $\mathbb{Z}/n\mathbb{Z}$ we used in class.

You are allowed to use wihtout proof the fact that if n is prime, then any non-zero class in $\mathbb{Z}_n$ has a multiplicative inverse.

1. Consider the equivalence relation on $\mathbb{R}$ given by $a \sim b$ if $a - b$ is an integer multiple of 360. Construct a bijection between the set of equivalence classes $\mathbb{R}/\sim$ and the unit circle in $\mathbb{R}^2$.

2. Let $[a] \in \mathbb{Z}_n$ be invertible: there is a class $[b] \in \mathbb{Z}_n$ such that $[a] \cdot [b] = [1]$. Prove that then $[a][x] = [a][y]$ if and only if $[x] = [y]$. Is it true without the assumpiton that $[a]$ is invertible?

3. Use pigeonhole principle to show that for any $[a] \in \mathbb{Z}_n$, there exist $k \neq l$ such that $[a]^k = [a]^l$. Deduce from this that if $[a]$ is invertible, then there exists a number $k > 0$ (period) such that $[a]^k = [1]$.

4. Find $[2]^{2016}$in $\mathbb{Z}_{11}$.

5. Textbook, p. 272, problem 16

6. Textbook, p. 273, problem 18