MAT 200: HOMEWORK 10
DUE WED, APRIL 27, 2016

1. For each of the following relations on $\mathbb{R}$, determine whether it is an equivalence relation. If it is, describe the equivalence class containing 0.5.
  (a) $x \sim y$ if $x - y \in \mathbb{Z}$
  (b) $x \sim y$ if $x + y \in \mathbb{Z}$
  (c) $x \sim y$ if $\sin(\pi x) = \sin(\pi y)$
  (d) $x \sim y$ if $x^2 + y^2 = 1$

2. Consider the relation on $\mathbb{Z}$ given by
  $m \sim n$ if $m - n$ is a multiple of 8.
  Prove that it is an equivalence relation, and that the set of equivalence classes $\mathbb{Z}/\sim$ is finite. List all elements of $A/\sim$.

3. Consider an equivalence relation on $\mathbb{R}^2$ given by
  $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
  (a) Describe the equivalence class containing pair $(1, 1)$.
  (b) Construct a bijection between the set of equivalence classes $\mathbb{R}^2/\sim$ and the set $\mathbb{R}$ or real numbers.

4. Let $A = \{(m, n) \mid m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$. Consider the relation on A given by $(m, n) \sim (k, l)$ if $ml = kn$.
  (a) Show that it is an equivalence relation. In your proof, use only properties of integer numbers.
  (b) Describe the equivalence class containing pair $(2, 3)$.
  (c) Construct a bijection between the set of equivalence classes $A/\sim$ and the set of rational numbers $\mathbb{Q}$.