MAT131 Fall 2010 Midterm 1

Name: ________________________________ SB ID number: ______________________

Please circle the number of your recitation.

1. TuTh 12:50 – SB Union Gianniotis	**6.** MW 10:40 – Physics Medina	**11.** TuTh 3:50 – SB Union Lee
3. TuTh 8:20 – SB Union Lee	**7.** MW 6:50 – Physics Atyam	**12.** TuTh 8:20 – SB Union Wroten
4. WF 11:45 – Lgt Engr Lab Boyd	**8.** MW 3:50 – Old Chem Medina	**13.** MF 12:50 – Physics Kim
5. MF 2:20 – Library Kim	**9.** TuTh 5:20 – SB Union Gianniotis	

**************************** **DO NOT WRITE BELOW THIS LINE.** *************

Problem 1: ______ /20	**Problem 3**: ______ /25	**Problem 5**: ______ /10
Problem 2: ______ /20	**Problem 4**: ______ /25	
		TOTAL: _________ /100

Instructions: The exam is closed book, closed notes, calculators are not allowed, and all cellphones and other electronic devices must be turned off for the duration of the exam. You will have approximately 90 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., then **please raise your hand**.

Name: ________________________________ **Problem 1**: ______________ /20

Problem 1(20 points) In each of the following statements, circle T if it is true and F if it is false. Each part is worth **only 2 points out of 100 total points**. Remember to use your time wisely. There is no need to show your work on this problem.

T F (a) If both $f(x)$ and $g(x)$ are continuous at $x = a$, then also $f(x)g(x)$ is continuous at $x = a$.

T F (b) If both $f(x)$ and $g(x)$ are discontinuous at $x = a$, then also $f(x)g(x)$ is discontinuous at $x = a$.

T F (c) Each function $f(x)$ which is everywhere continuous is also everywhere differentiable.

T F (d) An even function cannot have 2 different horizontal asymptotes.

T F (e) If $f(x)$ is even and differentiable, then the derivative $f'(x)$ is odd.

T F (f) The function $f(x) = x^3 + \cos(x)$ is zero for at least one real number x.

T F (g) $\lim_{x\to\infty} \tan(x) = \infty$.

T F (h) If $\lim_{x\to\infty} f(x)$ does not exist, then $\lim_{x\to\infty} f(x)$ equals either ∞ or $-\infty$.

T F (i) Let $f(x)$ be a continuous function defined on the interval $[a, b]$. Let L be a real number which is not between $f(a)$ and $f(b)$. Then by the Intermediate Value Theorem, there does not exist a number c between a and b such that $f(c) = L$.

T F (j) If the function $f(x)$ is everywhere defined and is invertible, and if $g(y)$ is everywhere defined and is invertible, then also $g(f(x))$ is everywhere defined and is invertible.

Name: ______________________________ **Problem 2**: ______________ /20

Problem 2(20 points) The function $f(x)$ is defined by the following formula.

$$f(x) = \frac{3x-5}{2-x}.$$

(a)(5 points) Find the vertical and horizontal asymptotes for $y = f(x)$. Remember to show work justifying your answers.

Vertical Asymptote. ____________ **Horizontal Asymptote.** ____________

(b)(5 points) Find a formula for the inverse function $f^{-1}(x)$. Show your work.

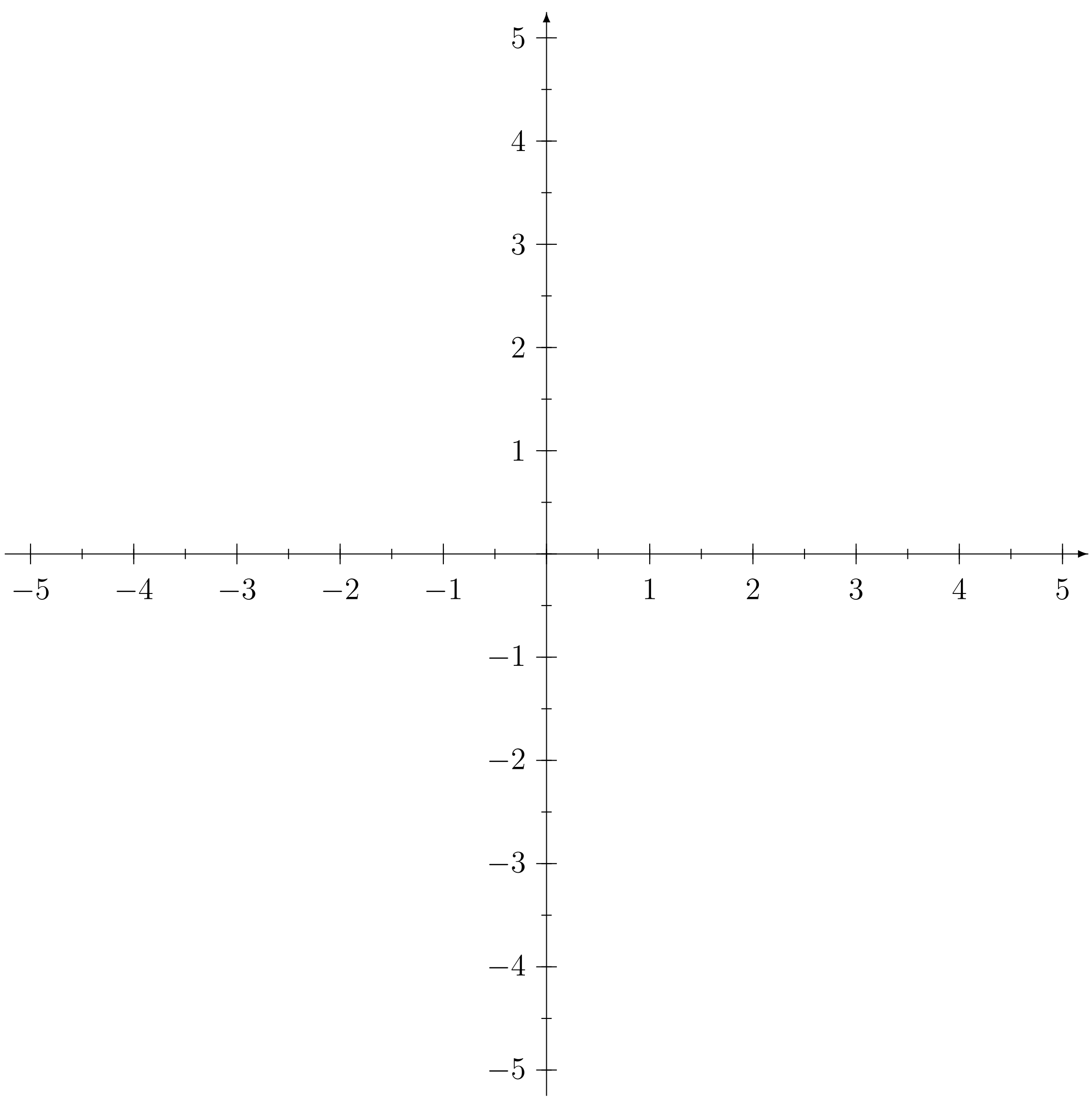
$\boldsymbol{f^{-1}(x) =}$ ____________

(c)(5 points) Find all vertical and horizontal asymptotes for $y = f^{-1}(x)$. Remember to show work justifying your answers.

Vertical Asymptote. ____________ **Horizontal Asymptote.** ____________

Name: ______________________________ **Problem 2, continued**

(d)(5 points) On the grid below, sketch the graph of both $y = f(x)$ and $y = f^{-1}(x)$. Each graph has **no** local maximum, **no** local minimum, and **no** inflection point. Carefully label all vertical and horizontal asymptotes of each curve. Your graph should make clear how each curve approaches each asymptote from each relevant side.



Name: ______________________________ **Problem 3**: ____________ /25

Problem 3(25 points) Consider the function $f(x) = -2 + 3\sqrt{x}$.

(a)(20 points) Use the limit definition of the derivative to compute $f'(4)$. **Remember to show all your work.**

(b)(5 points) Find the equation of the tangent line at $(4, f(4))$. Write your answer in **slope-intercept form**, $y = mx + b$.

$\boldsymbol{y} =$ ____________

Name: ____________________ **Problem 4**: ____________ /25

Problem 4(25 points) Consider the piecewise-defined function given by the following formula.

$$f(x) = \begin{cases} \sqrt{x^2-2x}-x, & x \geq 2 \\ \frac{|x|-2}{|x|-1} & x < 2 \text{ and } \mid x \mid \neq 1, \\ 3 & \mid x \mid = 1 \end{cases}$$

(a)(12 points) At each of the following points, circle **Cont.** if the function is continuous at that point, and circle **Discont.** if the function is discontinuous at that point. If the function is discontinuous, also circle the letter of the type of discontinuity: **R** for **R**emovable, **J** for **J**ump, or **I** for **I**nfinite. Show your work.

$\boldsymbol{x = +2}$. **Cont.** or **Discont.** If discont., the type is: **R** **J** **I**

$\boldsymbol{x = +1}$. **Cont.** or **Discont.** If discont., the type is: **R** **J** **I**

$\boldsymbol{x = 0}$. **Cont.** or **Discont.** If discont., the type is: **R** **J** **I**

$\boldsymbol{x = -1}$. **Cont.** or **Discont.** If discont., the type is: **R** **J** **I**

(c)(13 points) Compute both limits

$$\lim_{x\to+\infty} f(x) \text{ and } \lim_{x\to-\infty} f(x).$$

Then state the equations of all horizontal asymptotes. Show your work.

$\boldsymbol{\lim_{x\to+\infty} f(x) =}$ ____________

$\boldsymbol{\lim_{x\to-\infty} f(x) =}$ ____________

Equation of each horiz. asymptote. ____________________

Name: ___________________________________ **Problem 5**: _______________ /10

Problem 5(10 points) The following function is everywhere continuous.

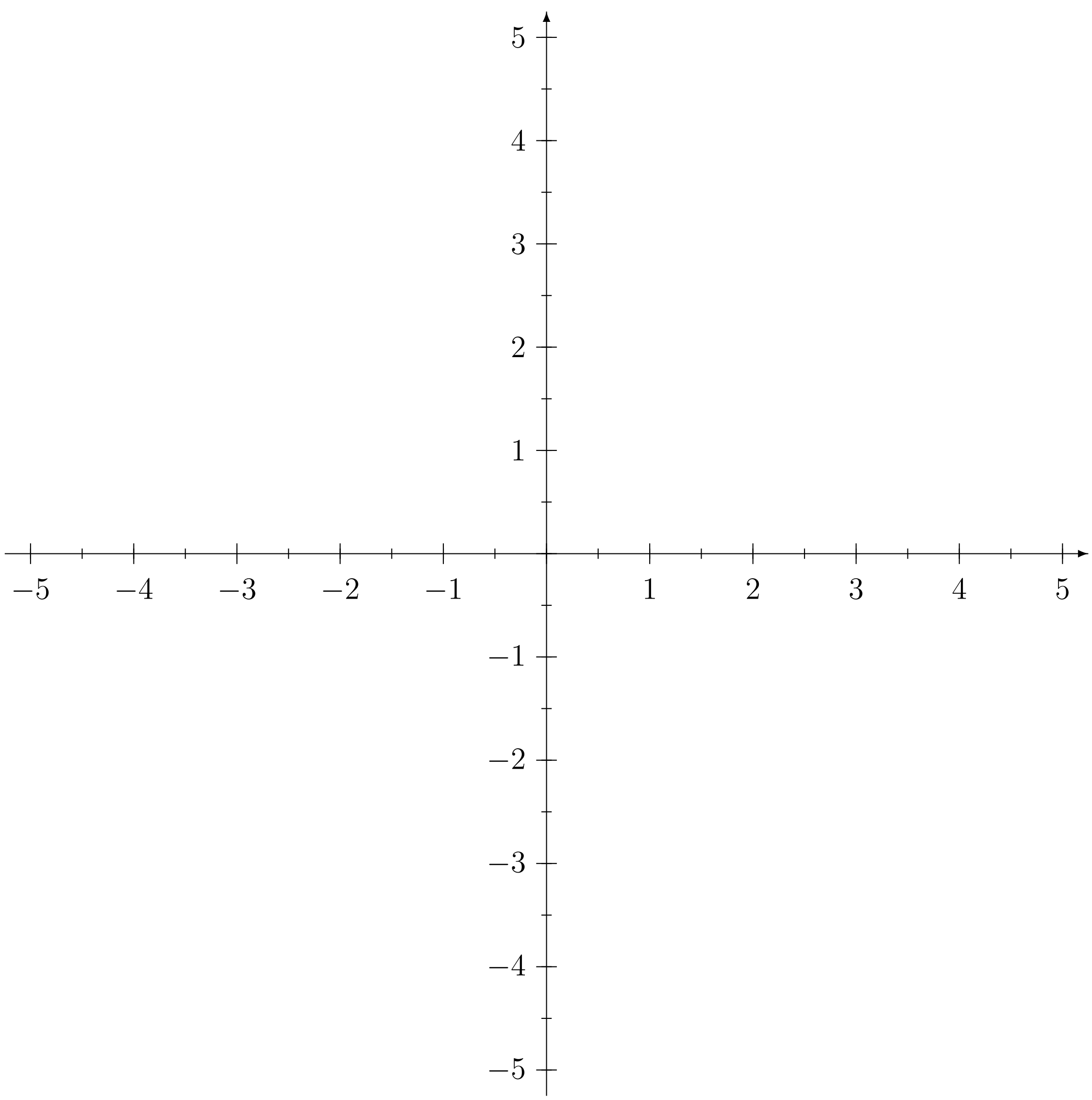
$$f(x) = \begin{cases} x^2, & x > 1 \\ 2 \mid x \mid -1, & x \leq 1. \end{cases}$$

(a)(5 points) At each of the following points, say whether the function is differentiable by circling the correct answer: **Diff.** if it is differentiable and **Nondiff.** if it is nondifferentiable. **Show your work and give reasons for your answers.** In this problem, you may use any formula you know for the derivatives; you need not use the limit definition.

$\boldsymbol{x = 2}$. **Diff.** **or** **Nondiff.**

$\boldsymbol{x = 0}$. **Diff.** **or** **Nondiff.**

(b)(5 points) On the grid below, sketch the graph of $y = f(x)$. Carefully label the points on the graph where $x = 0$ and where $x = 2$. Make certain your sketch matches your answer from (a) at each of these points.



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