MAT 127, MIDTERM 1
PRACTICE PROBLEMS

The midterm covers chapters 8.1 — 8.6 in the textbook. The actual exam will contain 5 problems (some multipart), so it will be shorter than this practice exam.

1. Determine whether the following sequence converges. If it converges, find the limit

$$a_n = (-1)^n \frac{n+4}{n^3 - 2n^2 + 4}$$

2. Determine whether the following sequence converges. If it converges, find the limit

$$a_n = \frac{3^n + 1}{n!}$$

3. Determine whether the following sequence converges. If it converges, find the limit

$$a_n = \frac{(\ln n)^2}{n}$$

4. Let the sequence a_n be defined by $a_1 = 1$, $a_{n+1} = \dfrac{3 + a_n}{2}$ for $n \geq 1$.

(a) Show that this sequence is bounded: $a_n \leq 3$ for all n.

(b) Explain why this sequence is convergent and find the limit.

5. If $\displaystyle\sum_{n=1}^{\infty} a_n$ is a convergent series with positive terms, what can you say about the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \sin(a_n)$? Does it converge? Does it converge absolutely?

6. For which values of p is the series $\displaystyle\sum_{n=1}^{\infty} p^n \frac{n!}{(2n)!}$ convergent?

7. If the series $\displaystyle\sum_{n=1}^{\infty} c_n 4^n$ is divergent, what can you say about the following series:

$$\sum_{n=1}^{\infty} c_n 2^n, \qquad \sum_{n=1}^{\infty} c_n (-8)^n, \qquad \sum_{n=1}^{\infty} c_n (-4)^n.$$

8. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1 + (-2)^n}{3^n}$$

9. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+3)}$$

10. Write the number $1.1\overline{009} = 1.1009009009\ldots$ as a fraction

11. Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{\sin(3\pi n/7)}{n^2+1}$$

12. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n \cdot 3^n}$$

Find the radius of convergence and the interval of convergence. You are not required to determine whether the series is convergent at the endpoints of the interval of convergence.

13. Write the function $f(x) = \ln(1+2x)/x$ as a power series in x. Find the radius of convergence of this series.