Problem. *Let f(x) and g(x) be two functions such that:*
$\int_{-1}^{2}[f(x)+g(x)]dx = 3, \int_{-1}^{2}[f(x)-2g(x)]dx = 1, \int_{-1}^{0} f(x)dx = -1$
Find $\int_0^2 f(x)dx$

Proof.

$$\int_0^2 f(x)dx = \int_{-1}^{2} f(x)dx - \int_{-1}^{0} f(x)dx \tag{1}$$
$$= \int_{-1}^{2} f(x)dx - (-1) \tag{2}$$

and we have:

$$\int_{-1}^{2} f(x)dx = \frac{1}{3}\left(2\int_{-1}^{2}[f(x)+g(x)]dx + \int_{-1}^{2}[f(x)-2g(x)]dx\right) \tag{3}$$
$$= \frac{1}{3}(2\cdot 3+1) \tag{4}$$
$$= \frac{7}{3} \tag{5}$$

so we can conclude that:
$\int_0^2 f(x)dx = \frac{7}{3}+1 = \frac{10}{3}$

Problem. *1) find* $\frac{d}{dx}\left(e^{x^2}\right)$
2)Evaluate
$\int_0^2 xe^{x^2}dx$

Proof. 1) we use the chain rule to get: $\frac{d}{dx}\left(e^{x^2}\right) = 2xe^{x^2}$

2) Note that by part (1), the antiderivative of xe^{x^2} is $\frac{1}{2}e^{x^2}$. Now apply the FUNDAMENTAL THEOREM OF CALCULUS to get:
$\int_0^2 xe^{x^2}dx = \frac{1}{2}e^{x^2}\Big]_0^2 = \frac{1}{2}\left(e^4 - e^0\right) = \frac{1}{2}\left(e^4-1\right)$

Problem. *Find the antiderivative of:*
1. $\frac{sin(2x)}{cos(x)}$
2. $e^{x+7}2^{-2x}$
3. $\frac{x^2}{x^3}$

Proof. 1. Use the 'double angle formula': $sin(2x) = 2sin(x)cos(x)$ and then the problem becomes VERY simple:
$\int \frac{sin(2x)}{cos(x)}dx = \int \frac{2sin(x)cos(x)}{cos(x)}dx = \int 2sin(x)dx = -2cos(x)+C$
2. So this problem seems like it may contain integration by part or substitution, but if we SIMPLIFY well we can avoid both:
$e^{x+7}2^{-2x} = e^7e^x\left(\frac{1}{4}\right)^x = e^7\left(\frac{e}{4}\right)^x$

Now there is only ONE function and it is NOT composite thus:

$$\int e^{x+7}2^{-2x}dx = \int e^7 \left(\frac{e}{4}\right)^x dx \tag{6}$$

$$= e^7 \int \left(\frac{e}{4}\right)^x dx \tag{7}$$

$$= e^7 \frac{1}{\ln\left(\frac{e}{4}\right)} \left(\frac{e}{4}\right)^x + C \tag{8}$$

$$= \frac{e^7}{1-\ln(4)} \left(\frac{e}{4}\right)^x + C \tag{9}$$

3. If there is one thing you should be noticing by now, it is this: TRY TO REDUCE THE FUNCTION BEFORE INTEGRATING OR DERIVATING! This last one is very simple one line calculation if we just notice that:

$\frac{x^2}{x^3} = x^{-1}$ (REDUCTION OF FRACTIONS HAS COME UP A LOT SO PAY ATTENTION TO IT!)

And so

$\int \frac{x^2}{x^3}dx = \int x^{-1}dx = \ln(x) + C$