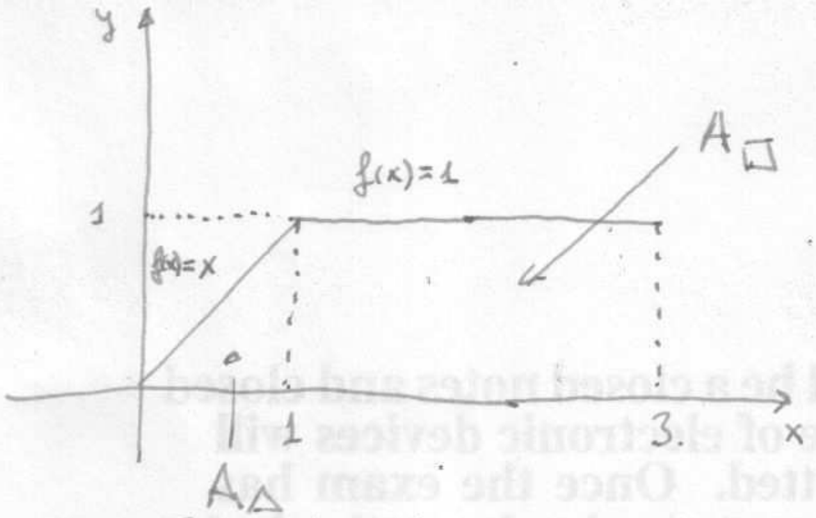
Problem 1 Evaluate the integral by interpreting it as an area:

$$\int_0^3 f(x)dx,$$

where

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 1, \\ 1 & \text{if } 1 \le x \le 3. \end{cases}$$

The graph of function is



The integral is the area A under the graph

$$A = A_{\triangle} + A_{\square}$$

$$A_{\triangle} = \frac{1}{2} \cdot \underbrace{1}_{\text{base of triangle}} \cdot \underbrace{1}_{\text{height of } \triangle} = \frac{1}{2}.$$

$$A_{\square} = 1 \cdot (3-1) = 2.$$

$$\int_0^3 f(x)dx = \frac{1}{2} + 2 = 2\frac{1}{2}.$$

Problem 2 Evaluate

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(4+\frac{5i}{n}\right)^{1/2}\frac{5}{n} \quad (\times)$$

by interpreting the limit as an integral and using the Evaluation Theorem to compute this integral.

Let $f:[a\,b]\to\mathbb{R}$ be a cont. function.

By definition

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f\left(a+\frac{b-a}{n}i\right) \quad (\times\times)$$

We compare two formulas $(\times)$ and $(\times\times)$

in our case the function $f(x)=x^{\frac{1}{2}}$.

$$4+\frac{5i}{n} = a+\frac{b-a}{n}i$$

We conclude that $4=a \quad b-a=5$

$$\Rightarrow b=5+a=5+4=9.$$

The limit is equal to

$$\int_a^b f(x)\,dx = \int_4^9 \sqrt{x}\,dx$$

Problem 3 Evaluate the following definite integral:

$$\int_2^4 \frac{x^3\sqrt{x^5} - x^2\sqrt[3]{x^2})}{x^4}dx$$

We simplify first

$$\frac{x^3\sqrt{x^5} - x^2\sqrt[3]{x^2}}{x^4} = x^{-4}\left(x^3 x^{\frac{5}{2}} - x^2 x^{\frac{2}{3}}\right) =$$

$$= x^{3+\frac{5}{2}-4} - x^{2+\frac{2}{3}-4} = x^{\frac{6+5-8}{2}} - x^{\frac{6+2-12}{3}} =$$

$$= x^{\frac{3}{2}} - x^{-\frac{4}{3}}$$

Antiderivative

$$\int_2^4 \left(x^{\frac{3}{2}} - x^{-\frac{4}{3}}\right)dx = \left(\frac{1}{\frac{3}{2}+1} x^{\frac{3}{2}+1} - \frac{1}{-\frac{4}{3}+1} x^{-\frac{4}{3}+1}\right)\Bigg|_2^4$$

$$= \left(\frac{2}{5} x^{\frac{5}{2}} + 3x^{-\frac{1}{3}}\right)\Bigg|_2^4 =$$

$$= \frac{2}{5} 4^{\frac{5}{2}} + 3(4)^{-\frac{1}{3}} - \left(\frac{2}{5} 2^{\frac{5}{2}} + 3\ 2^{-\frac{1}{3}}\right).$$