Practice Midterm 1

MAT 118

Feb 26, 2005

Name: (please print)	**ID #:**

Directions: There are 4 problems on 4 pages in this exam. Do all of your work in this exam booklet, and cross out any work that should be ignored. **Show your reasoning and computations — not just the answer.** You may not use any books, notes, calculators, or discussions with friends during this exam.

1. The eight members of a board of directors of an art museum must select a new museum head. There are three candiates: Zukoff (Z), Perella (P) and Mintz (M). The preference rankings of the board members are included in the table.

Number of voters	2	2	3	4
1^{st} place	Z	P	P	M
2^{nd} place	M	Z	M	P
3^{rd} place	P	M	Z	Z

(a) Which candidate would win a plurality election?
Solution: Z has 2 first place votes, P has 5, and M has 4, so P wins.

(b) Which candidate would win a plurality election with a runoff between the top two finishers?
Solution: The runoff will be between P and M; in the runoff, P would have 5 votes, and M will have 6 votes, so M will win.

(c) Which candidate would win the Borda count?
Solution: For Z: $2 \times 3 + 2 \times 2 + 7 \times 1 = 17$
For P: $2 \times 1 + 5 \times 3 + 4 \times 2 = 25$
For M: $2 \times 2 + 2 \times 1 + 3 \times 2 + 4 \times 3 = 24$
P wins.

(d) Which candidate, if any, is the Condorcet winner?
Solution:
In pairwise comparisons:
Z vs P: 2 for Z, 9 for P — P wins
P vs M: 6 for M, 5 for P — M wins
Z vs M: 5 for Z, 6 for M — M wins
Thus, M is the Condorcet winner

(e) Can the voters of the last group (those who ranked Mintz first) get a better result in Borda count method by voting strategically?
Solution: Yes: if they switch in their ballots P and Z (thus putting their preferences as M, then Z, then P), this will add 4 points to Z and subtract 4 points from P; then the point count would be
Z: 21
P: 21
M: 24
and M would win.

2. Consider the weighted voting system $[8 : 6, 4, 2, 1]$.

(a) Find the Banzhaf power index of each player.
Solution: Winning coalitions (with critical players underlined):

$$\{\underline{P_1}, \underline{P_2}\}, \quad \{\underline{P_1}, \underline{P_3}\}, \quad \{\underline{P_1}, P_2, P_3\}$$
$$\{\underline{P_1}, \underline{P_2}, P_4\}, \quad \{\underline{P_1}, \underline{P_3}, P_4\}, \quad \{\underline{P_1}, P_2, P_3, P_4\}$$

$B_1 = 6$, $B_2 = 2$, $B_3 = 2$, $B_4 = 0$, so $T = 6 + 2 + 2 = 10$, and
$\beta_1 = \frac{6}{10} = 60\%$, $\beta_2 = \beta_3 = \frac{2}{10} = 20\%$, $\beta_4 = 0$

(b) Are there any players who are dictators? have veto power? dummies? Justify your answer.
Solution: No player can form a winning coalition by himself, so there are no dictators. Every winning coalition must contain P_1, so he has veto power. Player P_4 is never critical, so he is a dummy player.

(c) Assume that player P_4 buys one vote from player P_1, so the new weights are $[8 : 5, 4, 2, 2]$. Find the new Banzhaf power index.
Solution:

$$\{\underline{P_1}, \underline{P_2}\}, \quad \{\underline{P_1}, \underline{P_2}, P_3\}, \quad \{\underline{P_1}, \underline{P_2}, P_4\}$$
$$\{\underline{P_1}, \underline{P_3}, \underline{P_4}\}, \quad \{\underline{P_1}, P_2, P_3, P_4\}, \quad \{\underline{P_2}, \underline{P_3}, \underline{P_4}\}$$

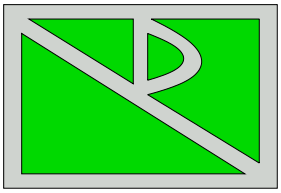
$B_1 = 5$, $B_2 = 4$, $B_3 = B_4 = 2$, so $T = 5 + 4 + 2 + 2 = 13$, and
$\beta_1 = \frac{5}{13} \approx 38\%$, $\beta_2 = \frac{4}{13} \approx 31\%$, $\beta_3 = \beta_4 = \frac{2}{13} \approx 15\%$

(d) Which would give player P_4 more power: buying one vote from P_1 or buying one vote from player P_2?
Solution: If P_4 buys one vote from P_2, the new weights would be $[8 : 6, 3, 2, 2]$, winning coalitions would be

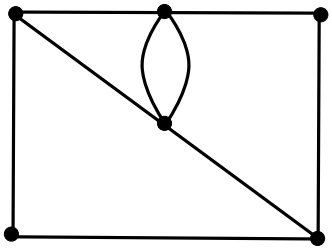
$$\{\underline{P_1}, \underline{P_2}\}, \quad \{\underline{P_1}, \underline{P_3}\}, \quad \{\underline{P_1}, \underline{P_4}\},$$
$$\{\underline{P_1}, P_2, P_3\}, \quad \{\underline{P_1}, P_2, P_4\}, \quad \{\underline{P_1}, P_3, P_4\}$$
$$\{\underline{P_1}, P_2, P_3, P_4\}$$

and the power index is $B_1 = 7$, $B_2 = B_3 = B_4 = 1$, $T = 10$, so $\beta_4 = \frac{1}{10} = 10\%$
Thus, it would give him less power than buying one vote from P_1

3. The sidewalks of a park are shown in the figure on the right (they go around the outside, across the middle, and make the D-shape). A city worker needs to sweep the walks, and wants to cover each one exactly once, without retracing his steps or walking on the grass.



(a) Draw the graph corresponding to this situation.



Solution:

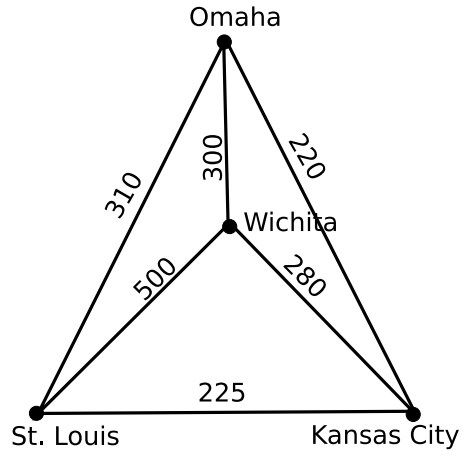
(b) Is it possible for the worker cover each part of the walk exactly once, ending up where he started? (That is, does the graph have an Eulerian circuit?) Justify your answer.
Solution: No: this graph has 2 vertices with odd number of edges. Namely, in the upper left corner and lower right corner, 3 walkways meet. So by Euler's theorem, no Eulerian circuit exists for this graph

(c) Is it possible for the worker cover each part of the walk exactly once if he doesn't need to end up where he started? (That is, does the graph have an Eulerian path?) Justify your answer.
Solution: Yes: since there are exaclty two odd vertices, by Euler's theorem it is possible to find a path which starts in one of them and ends in the other.

4. A person starting in Wichita must visit Kansas City, Omaha, and St. Louis (in any order), then return home to Wichita. You don't need to know that Omaha is north of Wichita, Kansas City is northeast, and St. Louis is east, but I'll tell you anyway. Approximate road mileages between the various cities are given below.

	Kansas City	**Omaha**	**St. Louis**	**Wichita**
Kansas City	—	220	225	280
Omaha	220	—	310	300
St. Louis	225	310	—	500
Wichita	280	300	500	—

(a) Draw a weighted graph which corresponds to the situation.



(b) Use the nearest neighbor algorithm to find an approximate solution to the traveling salesman problem, making a circuit starting at Wichita. What is the length of this circuit? Write your answers in the spaces below.
Solution: Wichita, Kansas City, Omaha, St. Louis, Wichita. Distance: 1310 miles

(c) Use the cheapest link algorithm to find an approximate solution to the traveling salesman problem, making a circuit starting at Wichita. What is the length of this circuit? Write your answers in the spaces below.

Wichita, St. Louis, Kansas City, Omaha, Wichita. Distance: 1245 miles

(d) Write the itinerary of the *shortest possible* solution to the traveling salesman problem in ths case.
Solution: By brute force, just trying all possible circuits, we see that there is a third possibility: a circuit in which Wichita is connected to Omaha and Kansas City. COmputing the length of this circuit, we see that it is indeed the shortest one:

Wichita, Omaha, St. Louis, Kansas City, Wichita. Distance: 1115 miles