MAT 127 Midterm II

November 3, 2010 8:30-10:00pm

Name:_____ **ID:**_____

first name last name

Section: L01 L02 L03 (circle yours)

MWF 9:35-10:30am TuTh 5:20-6:40pm TuTh 2:20-3:40pm

DO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(7) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(8) Be careful to avoid making grievous errors that are subject to heavy penalties.

(9) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1 (20pts)	2 (15pts)	3 (20pts)	4 (20pts)	5 (25pts)	Tot (100pts)

Problem 1 (20pts)

Answer Only: no explanation is required. Write your answer to each question in the corresponding box *in the simplest possible form.* No credit will be awarded if the answer in the box is wrong; partial credit may be awarded if the answer in the box is correct, but not in the simplest possible form. In (a)-(c), assume that the limits exist.

(a; 5pts) Find the limit of the sequence $a_n = \dfrac{\ln(64n^2+1) - \ln(n^2+n)}{4}$

(b; 5pts) Find the limit of the sequence $a_n = n\big(1-\mathrm{e}^{1/n}\big)$

(c; 5pts) Find the limit of the sequence

$$\sqrt{15},\ \sqrt{15+2\sqrt{15}},\ \sqrt{15+2\sqrt{15+2\sqrt{15}}},\ \sqrt{15+2\sqrt{15+2\sqrt{15+2\sqrt{15}}}},\ \ldots$$

(d; 5pts) Write the number $1.1\overline{45} = 1.1454545\ldots$ as a simple fraction

Problem 2 (15pts)

Determine whether each of the following sequences converges or diverges. In each case, *circle* your answer to the right of the question and *justify* it in the space provided below the question. You do *not* have to determine the limit if the sequence converges.

(a; 5pts) $a_n = \dfrac{(-1)^n\, n!}{n^n}$ (reminder: $n! = 1 \cdot 2 \cdot \ldots \cdot n$) **converge** **diverge**

(b; 5pts) $a_1 = 14, \quad a_{n+1} = \sqrt{7a_n}$ **converge** **diverge**

(c; 5pts) $a_1 = 1, \quad a_{n+1} = \sqrt{7 - 3a_n}$ **converge** **diverge**

Problem 3 (20pts)

(a; 10pts) Determine whether the following series **converges** or **diverges**

$$\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{4^n+1}}$$

Circle your answer above and justify it below.

(b; 10pts) Find all values of p for which the following series converges.

$$\sum_{n=1}^{\infty} n^p \sin^2(1/n)$$

Write your answer in the box to the right and justify it below.

Problem 4 (20pts)

For each of the following series,

(1) determine the corresponding sequence s_n of partial sums (sum of the first n terms);

(2) determine whether the series converges and if so, find it is sum.

Simplify your answers as much as possible and justify them.

(a; 8pts) $\displaystyle\sum_{n=1}^{\infty}(-1)^n$

(b; 12pts) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{4n^2-12n+5}$ *Hint:* partial fractions

Problem 5 (25pts)

A two-species interaction is modeled by the following system of differential equations

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = x - \frac{1}{500}x^2 - \frac{1}{40}xy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = y - \frac{1}{50}y^2 + \frac{1}{1000}xy \end{cases} \qquad (x, y) = (x(t), y(t)),$$

where t denotes time.

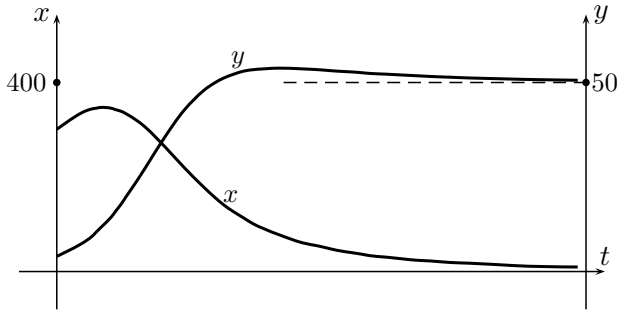
(a; 5pts) Which of the following best describes the interaction modeled by this system?

(i) predator-prey (ii) competition for same resources (iii) cooperation for mutual benefit

Circle your answer above and justify it below.

(b; 12pts) This system has 4 equilibrium (constant) solutions; find all of them and explain their significance relative to the interaction the system is modeling. **Answer Only:** put one equilibrium solution in each box below and use the space to the right of the box to describe its significance; use scrap paper or back side of a page in the exam to work out your answer.

(c; 8pts) The left diagram below shows the graphs of functions $x=x(t)$ and $y=y(t)$ so that the pair (x, y) solves the above system of differential equation. Sketch the corresponding (directed) phase trajectory on the right diagram below, adding appropriate markings to the axes and indicating coordinates of whatever points possible. Explain/indicate how you make your sketch!



(d; bonus 10pts, all or nothing) Show that one of the two species eventually goes extinct, according to this model (for any initial populations); use the facing page, the back of the exam, or an extra sheet.