MAT 127 Midterm I

October 6, 2010 8:30-10:00pm

Name: ______________________________ **ID:** ____________________

first name *first*

Section:	L01	L02	L03	(circle yours)
	MWF 9:35-10:30am	TuTh 5:20-6:40pm	TuTh 2:20-3:40pm	

DOO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(7) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(8) Be careful to avoid making grievous errors that are subject to heavy penalties.

(9) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

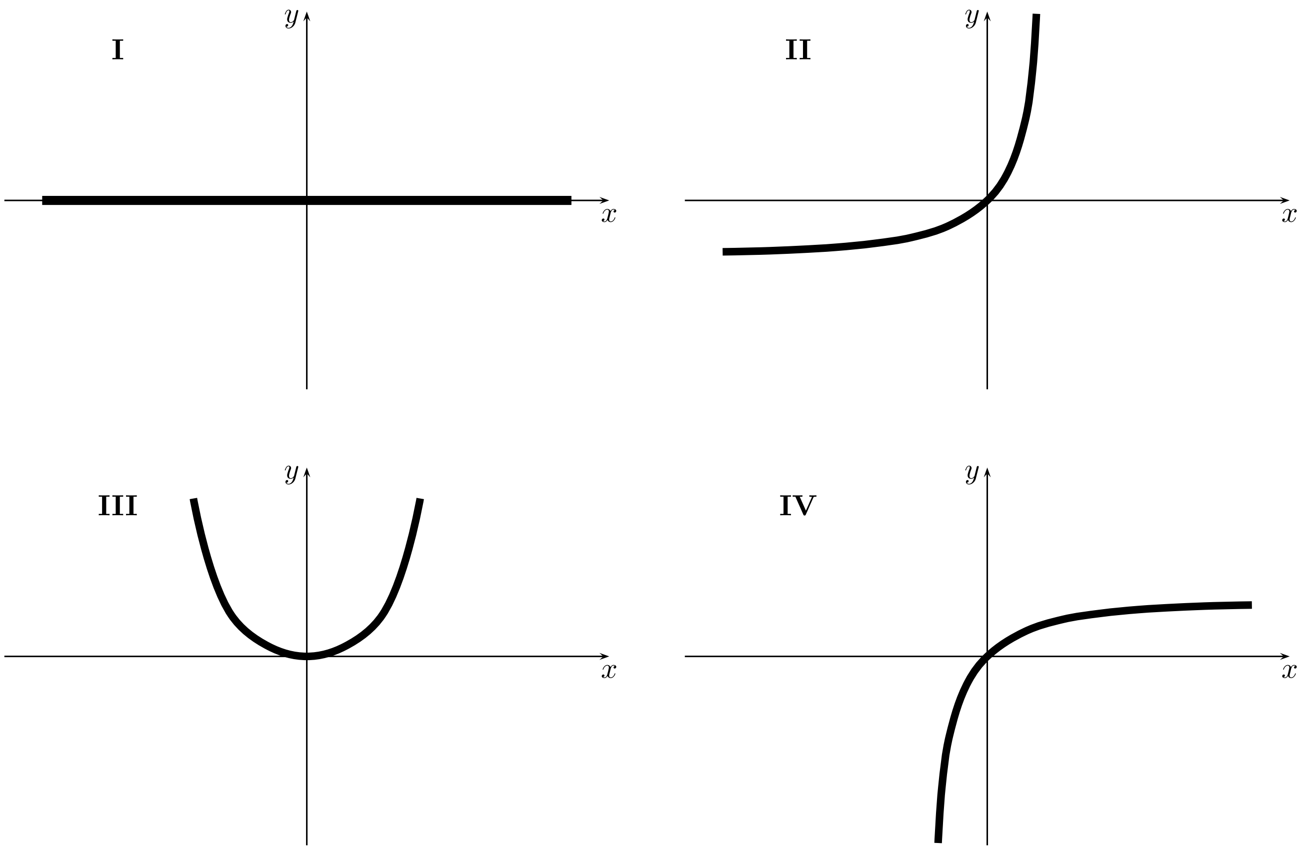
1 (10pts)	2 (15pts)	3 (15pts)	4 (20pts)	5 (20pts)	6 (20pts)	Tot (100pts)

Problem 1 (10pts)

Consider the four differential equations for $y = y(x)$:

(a) $y' = x(1+y^2)$ (b) $y' = y(1+x^2)$ (c) $y' = \mathrm{e}^{x+y}$ (d) $y' = \mathrm{e}^{-x-y}$.

Each of the four diagrams below shows a solution curve for one of these equations:



Match each of the diagrams to the corresponding differential equation (the match is one-to-one):

diagram	I	II	III	IV
equation				

Answer Only: no explanation is required.

do not write below this line or your work on this problem will be void

grader's use only

correct − repeats	0-	1	2	3	4
points	0	2	5	9	10

Problem 2 (15pts)

(a; 7pts) Show that the function $y(x) = x\mathrm{e}^{-2x}$ is a solution to the initial-value problem

$$y'' + 4y' + 4y = 0, \quad y = y(x), \qquad y(0) = 0, \qquad y'(0) = 1.$$

Show your work and/or explain your reasoning.

(b; 8pts) Find the general solution of the differential equation

$$y'' + 4y' + 4y = 0, \quad y = y(x).$$

Show your work and/or explain your reasoning.

Problem 3 (15pts)

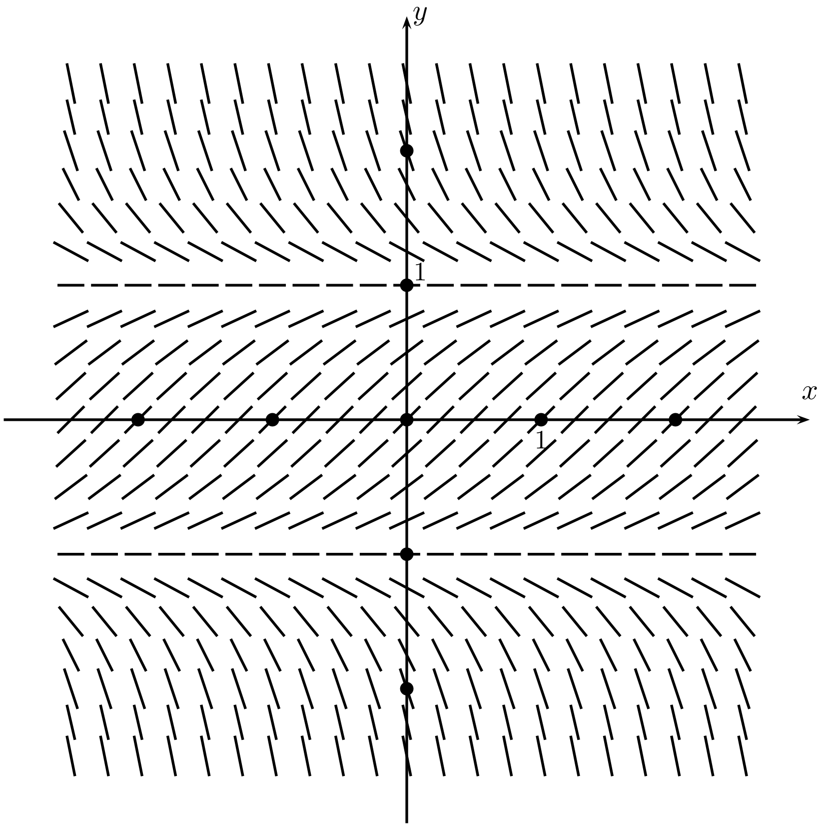
A sample of tritium-3 (a radioactive substance) decayed to 90% of its original amount in 2 years.

(a; 10pts) Let $y = y(t)$ be the ratio of the amount of tritium-3 remaining after t years to the original amount. Find a formula for $y(t)$. *Show your work and/or explain your reasoning.*

(b; 5pts) If the initial weight of the sample was 100 mg (at time $t = 0$), what is the weight of the sample after 6 years (from $t = 0$)? *Show your work and/or explain your reasoning.*

Problem 4 (20pts)

The direction field for a differential equation is shown below.



(a; 16pts) On the direction field, sketch and *clearly label* the graphs of the four solutions with the initial conditions $y(0) = -.5$, $y(0) = 0$, $y(0) = 1$, and $y(0) = 2$ (each of these four conditions determines a solution to the differential equation). *No explanation is required.*

(b; 4pts) The direction field above is for one of the following differential equations for $y = y(x)$:

$$\text{(i) } y' = 1 - y^2\,, \qquad \text{(ii) } y' = y^2 - 1\,, \qquad \text{(iii) } y' = y^4 - 1\,.$$

Which of these three equations does the direction field correspond to and why?

Problem 5 (20pts)

(a; 15pts) Let $y = y(x)$ be the solution to the initial-value problem

$$y' = x^2 - \frac{1}{3}y, \quad y = y(x), \qquad y(1) = 0.$$

Use Euler's method with $n = 3$ steps to estimate the value of $y(2)$. Show your steps clearly and use simple fractions (so $5/4$ or $\frac{5}{4}$, not 1.25).

(b; 5pts) Sketch the path in the xy-plane that represents the approximation carried out in part (a) and indicate its (path's) primary relation to the graph of the actual solution $y = y(x)$ of the initial-value problem in (a).

(c; bonus 5pts, all or nothing) Is your estimate for $y(2)$ in part (a) an *over*-estimate (larger than $y(2)$) or an *under*-estimate? Justify your answer; *no* credit for correct answer only. *Warning:* your time is likely to be better spent double- and triple-checking your work on the rest of the exam.

Problem 6 (20pts)

(a; 5pts) What are the constant solutions of the differential equation

$$y' = 2y(y-2), \qquad y = y(x) \ \ ?$$

Show your work and/or explain your reasoning.

(b; 15pts) Find the general solution to the differential equation

$$y' = 2y(y-2), \qquad y = y(x).$$

Simplify your answer as much as possible. *Show your work and/or explain your reasoning.*