Review Sheet: Test 1

The test will cover sections 1.1-1.6, and 3.1 (excluding the material on repeated roots) from the book, in addition to all material covered in the lectures up to and including 2/27. You may bring one sheet of paper (8.5in$\times$11in) to the test, on which you may record any helpful formulas, problems, etc... However, calculators and other computing devices will not be permitted. The following list of problems will help in preparing for the exam.

1. Find the general solution of

$$a)\ y' + y = xe^{-x} + 1, \qquad b)\ xy' = \sin x - 2y, \quad x > 0.$$

2. Find the solution of the given initial value problem and sketch

$$a)\ xy' + (x+1)y = x, \quad y(\ln 2) = 1, \qquad b)\ y' + \frac{2}{x}y = \frac{\cos x}{x^2}, \quad y(\pi) = 0, \quad x > 0.$$

3. Determine an interval in which the solution of the given initial value problem is certain to exist

$$a)\ (4 - x^2)y' + 2xy = 3x^2, \quad y(-3) = 1, \qquad b)\ y' + (\tan x)y = \sin x, \quad y(\pi) = 0.$$

4. Solve

$$a)\ y' = y^2\cos^2(2x), \qquad b)\ xy' = (1 - y^2)^{1/2}.$$

5. Determine the region in the xy-plane where the hypotheses of the existence and uniqueness theorem apply. Thus there is a unique solution through each given initial point in this region.

$$a)\ y' = \frac{\ln|xy|}{1 - x^2 + y^2}, \qquad b)\ y' = (x^2 + y^2)^{3/2}.$$

6. Radium-226 has a half-life of 1620 years. Find the time period during which a body of this material is reduced to three-fourths of its original size.

7. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has temperature of 200F when freshly poured, and one minute later has cooled to 190F in a room at 70F, determine when the coffee reaches a temperature of 150F.

8. A tank with a capacity of 500gal originally contains 200gal of water with 100lb of salt in solution. Water containing 1lb of salt per gallon is entering at a rate of 3gal/min, and the mixture is allowed to flow out of the tank at a rate of 2gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank

when it is at the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

9. An object of mass m is dropped from rest in a medium offering resistance proportional to the magnitude of the velocity. Find the time interval that elapses before the velocity of the object reaches 90% of its limiting value.

10. Determine whether or not each of the equations is exact. Find the solutions.

$$a)\ (2xy^2 + 2y) + (2x^2y + 2x)y' = 0, \qquad b)\ y' = e^{2x} + y - 1.$$

11. Use substitution to find the general solution.

$$a)\ xy' = y + \sqrt{x^2 + y^2}, \qquad b)\ xe^y y' = 2(e^y + x^3 e^{2x}).$$

12. Find the solution and sketch.

$$a)\ y'' + 4y' + 3y,\ \ y(0) = 2,\ \ y'(0) = -1, \qquad b)\ y'' + 3y' = 0,\ \ y(0) = -2,\ \ y'(0) = 3.$$

13. Determine the largest interval in which the given initial value problem is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

$$(x - 3)y'' + xy' + (\ln|x|)y = 0,\ \ y(1) = 0,\ \ y'(1) = 1.$$

14. Verify that y_1 and y_2 are solutions of the given equations. Do they constitute a fundamental set of solutions?

$$a)\ y'' - 2y' + y = 0,\ \ y_1 = e^x,\ \ y_2 = xe^x,$$
$$b)\ (1 - x\cot x)y'' - xy' + y = 0,\ \ 0 < x < \pi,\ \ y_1 = x,\ \ y_2 = \sin x.$$

15. Determine whether the given pair of functions is linearly independent or linearly dependent.

$$a)\ f(x) = 3x - 5,\ \ g(x) = 9x - 15, \qquad b)\ f(x) = x,\ \ g(x) = x^{-1},$$
$$c)\ f(x) = e^{3x},\ \ g(x) = e^{3(x-1)}.$$

16. Find the Wronskian of two solutions of the equation

$$(\cos x)y'' + (\sin x)y' - \frac{x^2 e^{\sin x}}{1 + x^4} y = 0.$$

17. Determine if $f(x) = x^2|x|$ and $g(x) = x^3$ are linearly independent or dependent on $-1 < x < 1$. Can f and g be solutions of an equation $y'' + p(x)y' + q(x)y = 0$ with p and q continuous on $-1 < x < 1$.

18. Solve

$$a)\ y'' + 4y = 0,\ \ y(0) = 0,\ \ y'(0) = 1, \qquad b)\ y'' - 2y' + 5y = 0,\ \ y(\pi/2) = 0,\ \ y'(\pi/2) = 2.$$