

Final Exam Solutions

$$1) y \sin x + x^2 e^y - y = C$$

$$2) t = 1620 \frac{\log 4/3}{\log 2}$$

$$3a) x(t) = \cos(\sqrt{5}t)$$

$$3b) \text{ frequency} = \frac{\sqrt{5}}{2\pi}, \text{ period} = \frac{2\pi}{\sqrt{5}}, \text{ amplitude} = 1.$$

$$3c) x_s(t) = \frac{3}{17} \sin 2t - \frac{12}{17} \cos 2t.$$

$$4a) \vec{x}(t) = c_1 e^{4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{4t} \left[t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1/3 \\ 0 \end{pmatrix} \right].$$

$$4b) \underline{\Psi}(t) = \begin{pmatrix} e^{4t} & (t-1/3)e^{4t} \\ -e^{4t} & -te^{4t} \end{pmatrix}.$$

$$5) \vec{x}(t) = \begin{pmatrix} 2/3 \\ 4/3 \end{pmatrix} + e^{-6t} \begin{pmatrix} 4/3 \\ -4/3 \end{pmatrix}$$

$$6) \vec{x}(t) = \begin{pmatrix} e^{-2t} & 2e^{5t} \\ -3e^{-2t} & e^{5t} \end{pmatrix} \begin{pmatrix} 3/7 \\ 64/49 \end{pmatrix} - (t+1/7)e^{-2t} \begin{pmatrix} 2/7 \\ 1/7 \end{pmatrix}$$