MAT 127: Calculus C, Spring 2015

Homework Assignment 10

due by 3pm on Wednesday, 04/15

Please read Section 8.5 in the textbook thoroughly before starting on the problem below.

Problem G

(a) Show that the series

$$g(z) = \sum_{n=1}^{\infty} \left(\frac{1}{z - n\pi} + \frac{1}{z + n\pi} \right)$$

converges for every $z \neq m\pi$ for any nonzero integer m and that $g(0)=0$.
Hint: combine the fractions and use the *Absolute Convergence Test*.

(b) The function

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1}{z - n\pi} + \frac{1}{z + n\pi} \right)$$

is thus well-defined for every $z \neq m\pi$ for any integer m. Show that

$$\lim_{z \longrightarrow 0} z f(z) = 1, \qquad f(-z) = -f(z), \qquad f(z+\pi) = f(z), \qquad f(\pi/2) = 0, \tag{1}$$

with the middle identities holding whenever either side is defined ($z \neq m\pi$ for any integer m).
Hint: use partial sums for the third equality; the other three are easy.

(c) What is the "simplest" function that satisfies all identities in (1)? (answer only)

This all leads to

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

as stated in Section 8.3; see solutions for more details.