MAT 127 Final Exam

May 13, 2015 11:15am-1:45pm

Name:______________________________ **ID:**____________________

first name *first*

Section:	L1	L2	L3	L4	L5	(circle yours)
	MWF 10-10:52am	MW 4-5:20pm	MWF 11-11:53am	TuTh 8-9:20am	TuTh 4-5:20pm	

DO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(7) Show your work; correct answers alone will receive only partial credit (unless noted otherwise).

Out of fairness to others, please **stop working** and close the exam as soon as the time is called.

Some Taylor Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{if } \ |x| < 1 \qquad\qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad\qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

to receive full credit, justify any other power series expansion you use

1 (10pts)	
2 (15pts)	
3ab (20pts)	
4abc+d (20pts)	
5 (20pts)	
Subtotal (85pts)	

6abc+d (10pts)	
7 (15pts)	
8 (10pts)	
9 (10pts)	
10 (20pts)	
Subtotal (65pts)	

Total (150pts)	

Problem 1 (10pts)

Answer Only. Determine whether each of the following sequences or series converges or not. In each case, *clearly* circle either **YES** or **NO**, but not both. Each correct answer is worth 2 points. You may use the blank space between the questions to figure out the answer, but no partial credit will be awarded and no justification is expected for your answers on this problem.

(a) the sequence $a_n = 2 - \dfrac{\cos n}{n^{1/2}}$ **YES** **NO**

(b) the sequence $a_n = n^3 \sin(2/n) - 2n^2$ **YES** **NO**

(c) the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 + (-1)^n}{n^3 + 1}$ **YES** **NO**

(d) the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{3n - 2}$ **YES** **NO**

(e) the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{2^n + 10^n}}$ **YES** **NO**

Problem 2 (15pts)

Answer Only. Put your answers to (a) and (b) below in the corresponding box *in the simplest possible form.* No credit will be awarded if the answer in the box is wrong; partial credit may be awarded if the answer in the box is correct, but not in the simplest possible form.

(a; 5pts) Write the number $1.1\overline{6} = 1.16666\ldots$ as a simple fraction

(b; 5pts) Find the limit of the sequence

$$1+2\sqrt{2}, \quad 1+2\sqrt{2+\sqrt{2}}, \quad 1+2\sqrt{2+\sqrt{2+\sqrt{2}}}, \quad 1+2\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}, \qquad \ldots.$$

Assume that this sequence converges.

(c; 5pts) Suppose the power series $\sum_{n=0}^{\infty} c_n(x+2)^n$ converges at $x=-4$ and diverges at $x=4$. What can be said about the convergence of this power series at the following values of x?

$x=-2$:	*converges*	*diverges*	*not enough info to tell*
$x=-1$:	*converges*	*diverges*	*not enough info to tell*
$x=0$:	*converges*	*diverges*	*not enough info to tell*
$x=2$:	*converges*	*diverges*	*not enough info to tell*
$x=6$:	*converges*	*diverges*	*not enough info to tell*

Circle one answer per line.

Problem 3 (20pts)

Find Taylor series expansions of the following functions around the given point. In each case, determine the radius of convergence of the resulting power series and its interval of convergence. Show your work.

(a; 10pts) $f(x) = x^3 - 6x$ around $x = 1$

(b; 10pts) $f(x) = \dfrac{x}{2 - x^3}$ around $x = 0$

Problem 4 (20pts)

(a; 8pts) Find the radius and interval of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n+1)!} x^{2n} .$$

(b; 4pts) Find $\displaystyle \lim_{x \longrightarrow 0} \frac{f(x)-1}{x^2}$

(c; 8pts) Find the Taylor series expansion for the function $g = g(x)$ given by

$$g(x) = \int_0^x u^2 f(u)\, \mathrm{d}u$$

around $x=0$. What are the radius and interval of convergence of this power series?

(d; bonus 10pts)* Find $g(\pi/3)$; use the facing page, the back of this page, or an extra sheet.

* this part is relatively hard and subject to harsh grading; do and double-check the rest of the exam first

Problem 5 (20pts)

All questions in this problem refer to the infinite series

$$\sum_{n=1}^{\infty} n\,\mathrm{e}^{-4n}$$

(a; 3pts) Explain why this series converges.

(b; 4pts) What is the minimal number of terms required to approximate the sum of this series with error less than 1/100? Justify your answer. You may use that $\mathrm{e} \approx 3$ for the purposes of part (b).

(c; 3pts) Based on your answer in part (b), estimate the sum of this series with error less than 1/100; simply your answer as much as possible, but leave it in terms of e. Is your estimate an under- or over-estimate for the sum? Explain why. (If you do not know how to do (b), take the answer to (b) to be 3).

(d; 10pts) Find the sum of the infinite series exactly.

Problem 6 (10pts)

All questions in this problem refer to the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+3)\,3^n}$$

(a; 3pts) Explain why this series converges.

(b; 4pts) What is the minimal number of terms required to approximate the sum of this series with error less than $1/100$? Justify your answer.

(c; 3pts) Based on your answer in part (b), estimate the sum of this series with error less than $1/100$; leave your answer as a simple fraction p/q for some integers p and q with no common factor. Is your estimate an under- or over-estimate for the sum? Explain why. (If you do not know how to do (b), take the answer to (b) to be 3).

(d; bonus 10pts)* Find the sum of the infinite series exactly.

* this part is relatively hard and subject to harsh grading; do and double-check the rest of the exam first

Problem 7 (15pts)

Find the general real solution to each of the following differential equations.

(a; 5pts) $y'' - 2y' + y = 0, \quad y = y(x)$

(b; 5pts) $y'' + 2y' + 3y = 0, \quad y = y(x)$

(c; 5pts) $y'' + 2y' - 3y = 0, \quad y = y(x)$

Problem 8 (10pts)

Let $y = y(x)$ be the solution to the initial-value problem

$$y' = y, \quad y = y(x), \qquad y(0) = 1.$$

Use Euler's method with $n = 2$ steps to estimate the value of $y(1)$. Show your steps clearly and use simple fractions (so $5/4$ or $\frac{5}{4}$, not 1.25).

Problem 9 (10pts)

Answer Only. A two-species interaction is modeled by the following system of differential equations

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = x - \frac{1}{10}x^2 + \frac{1}{40}xy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{2}y - \frac{1}{100}xy \end{cases} \qquad (x,y) = (x(t), y(t)),$$

where t denotes time.

(a; 2pts) Which of the following best describes the interaction modeled by this system?

(i) predator-prey (ii) competition for same resources (iii) cooperation for mutual benefit

Circle your answer above.

(b; 8pts) This system has 3 equilibrium (constant) solutions; find all of them and explain their significance relative to the interaction the system is modeling. *Put one equilibrium solution in each box* below and *use the space to the right of the box to describe its significance.*

$\boxed{}$ ____

$\boxed{}$ ____

$\boxed{}$ ____

You can use the space below (or elsewhere) to figure out your answers, but your score will based on your **answers above only**.

Problem 10 (20pts)

(a; 10pts) Show that the orthogonal trajectories to the family of curves $x = ky^2$ are described by the differential equation

$$y' = -\frac{2x}{y}\,, \qquad y = y(x).$$

(b; 7pts) Find the general solution to the differential equation stated in (a).

(c; 3pts) Sketch one representative of the original family of curves and one orthogonal trajectory on the same diagram; indicate clearly which is which.