

MATH 127

Final Exam

May 15, 2006

Name: _____ ID: _____ Lect: _____

Lec 1: Saveleyev, MWF 9:35 Lec 2: Sutherland, TuTh 2:20 Lec 4: Unal, TuTh 5:20

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	12	12	12	12	12	10	10	10	10	100
Score:											

There are 10 problems in this exam, on 10 pages. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it.

You may use a single sheet of letter-sized paper containing handwritten notes to do this exam. The sheet of notes **must** be turned in along with your exam. **Books, calculators, additional papers, and discussions with friends are not permitted.**

Problems without full justification (ie, "work") will not receive full credit, even for otherwise correct answers.

Leave all answers in exact form (that is, do *not* approximate π , square roots, and so on.)

You have 150 minutes to complete this exam.

Name: _____

Id: _____

1. 10 points According to the poem by Ogden Nash,

Big fleas have little fleas,
Upon their backs to bite 'em,
And little fleas have lesser fleas,
And so, ad infinitum.

Assume each flea has exactly one flea which bites it.
If the largest flea weighs 0.03 grams, and each flea is $\frac{1}{10}$ the weight of the flea it bites, what is the total weight of all the fleas?

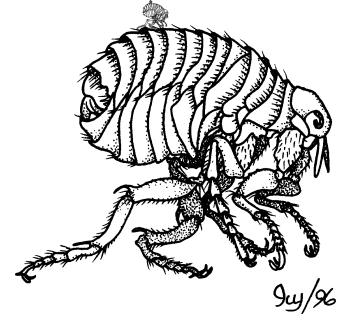


image adapted from
<http://bioidiac.bio.uottawa.ca>

Name: _____

Id: _____

2. 12 points Find the Maclaurin series of each of the given functions. I suggest you use a familiar power series as your starting point.

(a) e^{3x^2}

(b) $\frac{1}{1+4x}$

(c) $\sin 2x$

Name: _____

Id: _____

3. 12 points For each of the series below, decide if it converges or diverges. **You must justify your answer to receive full credit.** You do not need to find the sum of convergent series.

(a)
$$\sum_{n=1}^{\infty} \frac{2^n}{5^n + 3^n} = \frac{1}{4} + \frac{2}{17} + \frac{1}{19} + \frac{8}{353} + \dots$$

(b)
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} = \frac{1}{2 \ln 2} + \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} + \dots$$

(c)
$$\sum_{n=0}^{\infty} \frac{10^n}{n!} = 1 + 10 + 50 + \frac{500}{3} + \frac{1250}{3} + \dots$$

Name: _____

Id: _____

4. 12 points Let $f(x) = x^{1/3}$.

(a) Find the Taylor polynomial of degree 2 centered at $a = 8$ for $f(x)$.

(b) If the result of the previous part is used to estimate $9^{1/3}$, how large can the error be (according to Taylor's theorem)?

Name: _____

Id: _____

5. 12 points Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1)4^n}$$

Name: _____

Id: _____

6. 12 points Solve the initial value problem.

$$\frac{dy}{dx} = 6x^2y^2 \quad y(0) = 1$$

Name: _____

Id: _____

Do **any three** of problems 7, 8, 9, and 10. Cross out the one you don't want graded

7. 10 points Find the solution $y(t)$ to the initial value problem

$$y'' + 2y' - 3y = 0 \quad y(0) = 0 \quad y'(0) = 1$$

Name: _____

Id: _____

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8. 10 points A population of armadillos is well modelled by a logistic equation with a carrying capacity of 1000. Assume that initially there are 100 armadillos, so the equation is

$$P(t) = kP \left(1 - \frac{P}{1000} \right) \quad P(0) = 100$$

where t is in years and k is some constant.

- (a) Determine k if after one year there are 200 armadillos.

- (b) When will there be 500 armadillos?

Name: _____

Id: _____

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9. 10 points Consider the initial value problem

$$y'' + y' - xy = 0 \quad y(0) = 1 \quad y'(0) = 0$$

- (a) Use power series to find a degree 4 polynomial approximation to $y(x)$.

- (b) Is $y(0.5) > 1$? Justify your answer.

Name: _____

Id: _____

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10. 10 points A population of birds and insects is modelled by

$$\frac{dx}{dt} = 0.4x(1 - 0.000005x) - 0.002xy \quad \frac{dy}{dt} = -0.2y + 0.000008xy$$

where $x(t)$ is the number of insects and $y(t)$ is the number of birds.

- (a) Find **all** of the equilibrium solutions (also called “fixed points” or “constant solutions”). If there are none, write “None” and justify your answer.

- (b) The figure shows a phase trajectory starting with 10,000 insects and 100 birds. Describe what happens to the bird and insect populations as time passes.

