MAT 127 Midterm I

March 2, 2015 8:45-10:15pm

Name: _____ **ID:** _____

first name *first*

Section: L1 L2 L3 L4 L5 (circle yours)

MWF 10-10:52am | MW 4-5:20pm | MWF 11-11:53am | TuTh 8-9:20am | TuTh 4-5:20pm

DO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(7) Show your work; correct answers alone will receive only partial credit (unless noted otherwise).

Out of fairness to others, please **stop working** and close the exam as soon as the time is called.

Do not write in the boxes at the bottom of this page.

1 (15pts)	2 (15pts)	3 (15pts)	4 (15pts)	5 (20pts)	6 (20pts)	Tot (100pts)

Problem 1 (15pts)

Solve the following initial value problem

$$y' = \frac{x}{y^2}e^x, \qquad y(0) = 0.$$

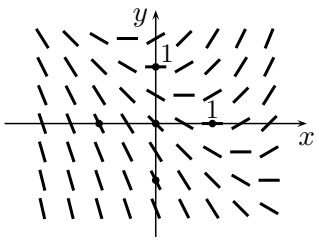
Problem 2 (15pts)

Consider the four differential equations for $y = y(x)$:

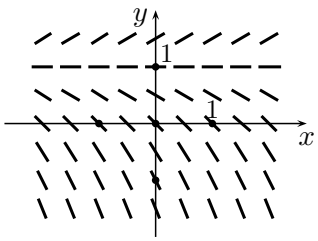
$$\text{(a) } y' = y - 1, \qquad \text{(b) } y' = y^2 - 1, \qquad \text{(c) } y' = x(y^2 - 1) \qquad \text{(d) } y' = x + y - 1.$$

Each of the two diagrams below shows the direction field for one of these equations:

I

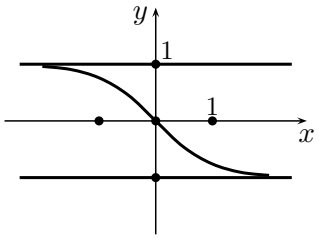


II

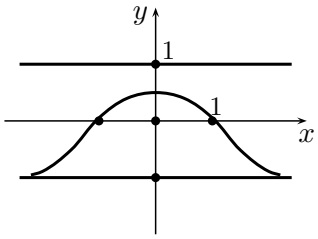


Similarly, each of the two diagrams below shows three solution curves for one of these equations:

III



IV



Match each of the diagrams to the corresponding differential equation (the match is one-to-one):

diagram	I	II	III	IV
equation				

Answer Only: no explanation is required.

Note, however, that you will be **penalized** for repeated answers. For example, if you answer (a) for both diagrams I and II, then points will be deducted.

Problem 3 (15pts)

A radioactive substance, Strontium-90, has a half-life of 30 days. A sample of Strontium-90 initially contains 80 mg. Let $y = y(t)$ denote the amount of this substance remaining after t days.

(a; 10pts) Find a differential equation satisfied by $y(t)$.

(b; 5pts) How much Strontium-90 will remain after 60 days?

Problem 4 (15pts)

Consider the autonomous differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (y+2)^2(y-1)(y-2).$$

(a; 5pts) Find the equilibrium (or constant) solutions.

(b; 10pts) If $y(0) = 0$, compute the following limit $\lim_{x\to\infty} y(x)$.

Problem 5 (20pts)

Alice and Bob each have an investment of \$1000 initially in accounts that compound interest continuously. Alice's investment doubles after $20 \ln 2$ days, and Bob's investment triples after $10 \ln 3$ days. Which account has more money after one year?

Problem 6 (20pts)

A tank contains 10 kg of salt dissolved in 2000 L of water. Brine that contains 0.04 kg of salt per liter of water enters that tank at a rate of 25 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate.

(a; 10pts) If $y(t)$ denotes the amount of salt in the tank at time t (measured in minutes), find a differential equation satisfied by y.

Hint: The total rate of change of the amount of salt in the tank is equal to the rate of salt going in minus the rate of salt going out.

(b; 10pts) How much salt remains in the tank after one hour?