NAME AND ID Number:

| Problem | 1 | 2 | 3 | Total |
| ---: | ---: | ---: | ---: | ---: |
| Score |  |  |  |  |

# Practice Midterm 2 

April, 2020
Total maximal score $=30$.
Show all your work. Write proofs carefully, clearly and COMPLETELY, SO THAT THEY CAN BE USED, FOR EXAMPLE, TO SATISFY the "Proof Writing" Mathematics Department graduation reQUIREMENT.
(1) (10 points) Suppose the sequence $\left(x_{n}\right)$ is defined by

$$
x_{1}=2, x_{n+1}=2-\frac{1}{x_{n}}
$$

for $n>0$. Show that $\left(x_{n}\right)$ is bounded and monotone. Find the limit.
(2) (10 points) Can you give an example of a convergent series $\Sigma x_{n}$ and a divergent series $\Sigma y_{n}$ such that $\Sigma\left(x_{n}+y_{n}\right)$ is convergent? Explain.
(3) (10 points) Determine and prove if the following series converge or diverge:
(a) $\sum_{n=1}^{\infty} \frac{n+1}{3 n+5}$
(b) $\sum_{n=1}^{\infty} \frac{n+1}{n^{3}+1}$

End of Examination.

