MAT535 HW 10

Due on 4/26 in class. Each problem is worth 10 points. You are required to do four of the problems of your choice. You can do the rest for extra credit.

Problem 1. Show that $I(\mathbb{A}^n) = (0)$.

Problem 2. If $I \subset R$ is any ideal, show that $\sqrt{I} :=$ rad I is a radical ideal.

Problem 3. Prove that:
(a) $S \subset I(Z(S))$.

(b) $W \subset Z(I(W))$.

(c) If W is an algebraic set, then $W = Z(I(W))$.

(d) If $I \subset k[x_1, \ldots, x_n]$ is any ideal, then $Z(I) = Z(\sqrt{I})$ and $\sqrt{I} \subset I(Z(I))$.

Problem 4. (a) Show that the set $X = \{(t, t^2, t^3) \in \mathbb{A}^3 | t \in k\}$ is closed in $\mathbb{A}^3$ and find $I(X)$.

(b) Same for the subset $Y = \{(t^3, t^4, t^5) \in \mathbb{A}^3 | t \in k\}$ of $\mathbb{A}^3$.

(c) Show that $I(Y)$ can't be generated by less than three polynomials.
Hint: Is $I(Y)$ a graded ideal ?

Problem 5. Show that $W = \{(x, y, z) \in \mathbb{A}^3 | x^2 = y^3, y^2 = z^3\}$ is an irreducible closed subset of $\mathbb{A}^3$ and find $I(W)$.

Problem 6. Find the radical $\sqrt{(y^2 + 2xy^2 + x^2 - x^4, x^2 - x^3)}$.

Problem 7. Let X be a Noetherian topological space. Prove that:

(a) If an irreducible closed set Y is contained in a union $\cup X_i$ of finitely many closed sets X_i, then $Y \subset X_i$ for some i.

(b) X has finitely many components.

(c) X is the union of its components.

(d) X is not the union of any proper subset of its components.