MAT536 HW6

Due 10/22 in class. Each problem is worth 10 points

Problem 1. Give examples of varieties X and Y, a point $P \in X$ and a morphism $\phi : X \setminus \{P\} \to Y$ such that ϕ can't be extended to a morphism on all of X in each of the cases:
(a) X is a non-singular curve and Y is not projective.
(b) X is a curve, P is a singular point and Y is projective.
(c) X is non-singular of dimension at least two, Y is projective.

Problem 2. Let X and Y be curves and $\phi : X \to Y$ a birational morphism.
(a) X_{sing} is a proper closed subset of X.
(b) $\phi(X_{sing}) \subseteq Y_{sing}$.
(c) If $y \in Y$ is a non-singular point, then $\phi^{-1}(y)$ contains at most one point.

Problem 3. Two non-singular projective curves are isomorphic if and only if they have the same function field.

Problem 4. *Resolution of singularities for curves.* Let X be a curve with smooth locus $U = X - X_{sing}$. Prove that there exists a non-singular curve $\tilde{X}$ with a surjective morphism $\phi : \tilde{X} \to X$ such that the restriction $\phi : \phi^{-1}(U) \to U$ is an isomorphism.

Problem 5. Let $E = V(y^2 - x^3 + x) \subset \mathbb{A}^2$. Show that if $P \in E$ is any point, then $E \setminus \{P\}$ is affine.

Problem 6. Let $E = V(y^2 - x^3 + x) \subset \mathbb{A}^2$, char $k \neq 2$. Prove that:
(a) E is a non-singular curve;
(b) the units in $k[X]$ are the non-zero elements of k;
(c) $k[X]$ is not a UFD;
(d) E is not rational.