MAT536 HW2

Due 9/24 in class. Each problem is worth 10 points

Problem 1. Let X be any space with functions and $Y \subset \mathbb{A}^n$ an affine variety. Show that a function $f : X \to Y$ is a morphism if and only if each coordinate function $f_i : X \to k$ is regular for $1 \leq i \leq n$.

Problem 2. Let $X = V(xy - zw) \subset \mathbb{A}^4$ and $U = D(y) \cup D(w) \subset X$. Define a regular function $f : U \to k$ by $f = \frac{x}{w}$ on $D(w)$ and $f = \frac{z}{y}$ on $D(y)$. Show that there are no polynomial functions $p, q \in A(X)$ such that $q(a) \neq 0$ and $f(a) = \frac{p(a)}{q(a)}$ for all $a \in U$.

Problem 3. Let X be an affine variety such that the affine coordinate ring $A(X)$ is a unique factorization domain. Let $U \subset X$ be an open subset. Show that if $f : U \to k$ is any regular function, then there exist $p, q \in A(X)$ such that $q(x) \neq 0$ and $f(x) = \frac{p(x)}{q(x)}$ for all $x \in U$.

Problem 4. (a) $k[\mathbb{A}^n \setminus \{0\}] = k[x_1, \ldots, x_n]$ for $n \geq 2$.
(b) $\mathbb{A}^n \setminus \{0\}$ is not an affine variety for $n \geq 2$.
(c) Every global regular function on $\mathbb{P}^n$ is constant, i.e. $k[\mathbb{P}^n] = k$
(d) $\mathbb{P}^n$ is not quasi-affine for $n \geq 1$.

Problem 5. Let $\phi : \mathbb{A}^1 \to V(y^2 - x^3) \subset \mathbb{A}^2$ be a morphism given by $\phi(t) = (t^2, t^3)$. Show that ϕ is not an isomorphism.

Problem 6. Let $X \subset \mathbb{P}^n$ be a projective variety with projective coordinate ring $R = k[x_0, \ldots, x_n]/I(X)$. Let $f \in R$ be a non-constant homogeneous element. Show that $D_+(f) \subset X$ is an open affine subvariety with affine coordinate ring $k[D_+(f)] = R_{(f)}$.