ERRATUM: SYMPLECTIC INVOLUTIONS OF $K3^{[n]}$ TYPE AND KUMMER N TYPE MANIFOLDS

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Abstract. In this note we present a corrected formula for the enumeration of connected components of the locus fixed by a symplectic involution inside hyperkähler manifolds of types $K3^{[n]}$ and generalized Kummer. We also provide further precisions concerning the involutions considered in the Kummer case.

In the paper [3] the enumerative part of Theorems 1.1 and 1.3 is incorrect. The source of error is an erroneous interpretation of the results of [1] in the proof of Lemma B.1. The error does not affect the enumeration of the top dimensional components as in Theorem 3.7 and 3.9. Moreover, the result of Theorem 1.3 does not apply to all involutions with a non trivial action on H^3, and we detail here the correct type of involutions considered.

The correct interpretation of the results of [1] yields the following statement

$$((\mathbb{C}^2)^{[n]})^\iota = \bigcup_{n_1+n_2=n} \mathfrak{M}_{Q'_0}((n_1,n_2),(0,1)),$$

where $n_1 \geqslant 0, n_2 > 0$ and $\mathfrak{M}_{Q'_0}((n_1,n_2),(0,1)) \neq \varnothing$ if and only if $d(n_1,n_2) = n_2 - (n_1 - n_2)^2 \geqslant 0$. In [3] we claim in the proof of Lemma B.1 that $\mathfrak{M}_{Q'_0}((n_1,n_2),(0,1)) = \varnothing$ if $|n_1 - n_2| > 1$, however this statement is false. Thus the count of the lower-dimensional components fixed by the involution is also incorrect.

The lower-dimensional component count is corrected in [4]. The special cases of Theorems 3.0.3 and 3.0.8 from [4] imply the correct count for the case considered in [3].

In the generalized Kummer case, in [3, Theorem 1.3] it was claimed that the result applies to all involutions with a non trivial action on H^3, however that is not the case, and the amended version of [3, Theorem 1.3] only applies to the second of the three following types of involutions.

Involutions on generalized Kummer manifolds can be subdivided in the following three categories:

- Involutions obtained by a translation by a two torsion point.
- Involutions obtained by a sign change composed with a translation by a two torsion point.
- Involutions obtained by a composition of a translation by a point of order at least three with a sign change.

The first kind of involutions only appears when n is odd, and they are the only ones with trivial action on H^3.

Our result only applies to the second kind of involutions, and we call them *regular* involutions. The mistake is contained in [3, Corollary 3.4], where the third kind of involutions was not considered. The fixed locus of the first kind of involutions was analyzed in [5, Lemma 3.5], while the third kind was analyzed only in the case of Kummer sixfolds in [2, Lemma 2.4]. It would be interesting to compute the fixed locus in general for an involution of the third kind.

Remark 1. All three kinds of involutions of Generalized Kummer manifolds can be recognized by their action on higher cohomology: involutions obtained by pure translations have a trivial action on H^3, and involutions of the third kind, those obtained by composing a translation of order at least three with -1, are recognizable by their action on higher cohomology. As an example, if $n \equiv 3$ modulo 4, in H^6 there are 256 distinguished classes of codimension three subvarieties of a generalized Kummer. These subvarieties are in correspondance with points in $A[4]$, each of them is the locus of subschemes having support of multiplicity at least four on a given four torsion point of A. The involutions we consider fix 16 of these subvarieties and permute the rest, while involutions of the third kind (obtained with an order four translation) freely permute these subvarieties.

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